FUZZY INTERPOLATION RATIONAL BICUBIC BEZIER SURFACE

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ABSTRACT. *This paper introduces fuzzy interpolation rational bicubic Bezier surface (later known as FIRBBS) which can be used to model the fuzzy data forms after defining uncertainty data by using fuzzy set theory. The construction of FIRBBS is based on the definition of fuzzy number concept since we dealing with the real uncertainty data form and interpolation rational bicubic Bezier surface model. Then, in order to obtain the crisp fuzzy solution, we applied the alpha-cut operation of triangular fuzzy number to reduce the fuzzy interval among those fuzzy data points(FDPs). After that, we applied defuzzification method to give us the final solution of getting single surface which also knows as crisp fuzzy solution surface. The practical example also is given which represented by figures for each processes. This practical example take the fuzzy data of lakebed modeling based on uncertainty at z-axis(depth).*

KEYWORD. Fuzzy number, rational bicubic Bezier surface, interpolation, alpha-cut, defuzzification.

INTRODUCTION

In Computer Aided Geometric Design (CAGD), rational spline function is one of the methods used in representing a set of data in the form of curves and surfaces. The advantage of rational spline function is that it has additional shape parameters known as weights. These weights provides extra freedom to the users to control the shape of the desired curves and surfaces (Farin 2002, 1999). Sarfraz et al, (1998) introduced a piecewise rational cubic spline curves where it possesses the weights in each interval to control the shape of the curves and surfaces. The extension of this piecewise rational cubic function is the rational bicubic function which was later introduced by Hussain and Hussain (2006b; 2007; 2006a).

The interpolation of piecewise rational method is one of a method in CAGD used in representing data especially in designing curves and surfaces. The representation of data through surface becomes difficult when the data is vague and imprecise due to various reason. This uncertainty data cannot be modeled directly with the stated methods to uncertainty. However, fuzzy set theory which was introduced by Zadeh (1965) can be used as the solution in defining data uncertainty. Fuzzy set theory is developed based on the concepts of fuzzy numbers which used to define the uncertainty level in real number (Klir et al, 1997; Klir and Yuan 1995). This research extended to combine fuzzy set theory and curves and surface function to achieve hybrid methods with the purpose of modeling uncertainty data via curves and surface (Zakaria and Wahab, 2013).

The structure first section of this paper discusses the theory of fuzzy sets in CAGD perspective. It is then, followed by the development of Fuzzy Interpolation Rational Bicubic Bezier Surface (FIRBBS) model. The effect of fuzzy weight variation and its corresponding shapes of FIRBBS are also discussed. Next, the operation of alpha-cut applied towards FIRBBS is discussed and explained with the aid of figures. Finally, the defuzzification method used to obtain the crisp fuzzy surface after the operation of alpha-cut against FIRBBS is discussed.

MATERIAL AND METHODS

This section introduces the fundamentals definitions which are used to construct FIRBBS. The readers are referred to (Zimmermann, 1985; Zakaria and Wahab, 2014; Zakaria et al, 2014) for a detailed explanation on the definitions. Short definitions are given below to ease the reader's understanding.

Definition 1. Let R be a universal set where R is a real number and A is subset to R. Fuzzy set, $\vec{A}$ in R (number around A in R) called fuzzy number which can be explained through the α-level set(strong and normal α-cut) that is if for every $\alpha \in (0,1]$, there exist set A_α in R with $\vec{A}_\alpha = \{x \in R : \mu_{A_\alpha}(x) > \alpha\}$ and $\vec{A}_\alpha = \{x \in R : \mu_{A_\alpha}(x) \geq \alpha\}$. (Zakaria, Wahab & Gobithaasan, 2015; Wahab & Zakaria, 2015).

Definition 2. If triangular fuzzy number is represented as $\vec{A} = (a,d,c)$ and $\vec{A}_\alpha$ is a α-cut operation of triangular fuzzy number, where the crisp interval by α-cut operation is obtained as $\vec{A}_\alpha = \left[a^\alpha, c^\alpha\right] = \left[(d-a)\alpha + a, -(c-d)\alpha + c\right]$ with $\alpha \in (0,1]$ where the membership function, $\mu_{\vec{A}}(x)$ given by

$$\mu_{\vec{A}}(x) = \begin{cases} 0 & \text{for } x < a \\ \dfrac{x-a}{d-a} & \text{for } a \leq x \leq d \\ \dfrac{c-x}{c-d} & \text{for } d \leq x \leq c \\ 0 & \text{for } x > c \end{cases} \tag{1}$$

In order to define the uncertainty real data problem, we used the definition of fuzzy relation to represent fuzzy data. Therefore, the definitions of fuzzy relation and fuzzy data are given as follow. (Zakaria, Wahab & Gobithaasan, 2015)

Definition 3. Let $X, Y \subseteq R$ be universal sets then

$$\vec{R} = \left\{((x,y), \mu_{\vec{R}}(x,y)) \,|\, (x,y) \subseteq X \times Y\right\}$$

is called a fuzzy relation on $X \times Y$ where $\times$ represents multiplication operation. (Zakaria, Wahab & Gobithaasan, 2015)

Definition 4. Let $X, Y \subseteq R$ and $\vec{A} = \left\{(x, \mu_{\vec{A}}(x) \,|\, x \in X\right\}$ and $\vec{B} = \left\{(y, \mu_{\vec{B}}(y) \,|\, y \in Y\right\}$ are two fuzzy sets. Then $\vec{R} = \left\{\left[(x,y), \mu_{\vec{R}}(x,y)\right], (x,y) \in X \times Y\right\}$ is a fuzzy relation on $\vec{A}$ and $\vec{B}$ if $\mu_{\vec{R}}(x,y) \le \mu_{\vec{A}}(x), \quad \forall (x,y) \in X \times Y$ and $\mu_{\vec{R}}(x,y) \le \mu_{\vec{B}}(y), \quad \forall (x,y) \in X \times Y$. (Zakaria, Wahab & Gobithaasan, 2015)

Definition 5. Let $X, Y \subseteq R$ and $\vec{M} = \left\{(x, \mu_{\vec{M}}(x) \,|\, x \in X\right\}$ and $\vec{N} = \left\{(y, \mu_{\vec{N}}(y) \,|\, y \in Y\right\}$ are two fuzzy data. Then, the fuzzy relation between both fuzzy data are given by $\vec{P} = \left\{\left[(x,y), \mu_{\vec{P}}(x,y)\right], (x,y) \in X \times Y\right\}$.(Zakaria, Wahab & Gobithaasan, 2015)

Definition 6. Let $P = \left\{(x,y),\ x \in X,\ y \in Y \,|\, x \text{ and } y \text{ are fuzzy data}\right\}$ and $\vec{P} = \left\{P_i \,|\, P \text{ is data point}\right\}$ are the set of FDPs which is $P_i \in P \subset X \times Y \subseteq R$ with R is the universal set and $\mu_P(P_i): P \to [0,1]$ is a membership function defined as $\mu_P(P_i) = 1$ which can be formulated as $\vec{P} = \left\{\left(P_i, \mu_P(P_i)\right) \,|\, P_i \in R\right\}$. Therefore,

$$\mu_P(P_i) = \begin{cases} 0 & \text{if and only if} \quad P_i \notin R \\ c \in (0,1) & \text{if and only if} \quad P_i \,\tilde{\in}\, R \\ 1 & \text{if and only if} \quad P_i \in R \end{cases} \tag{2}$$

with $\mu_P(P_i) = \left\langle \mu_P(P_i^{\leftarrow}), \mu_P(P_i), \mu_P(P_i^{\rightarrow})\right\rangle$ where $\mu_P(P_i^{\leftarrow})$ and $\mu_P(P_i^{\rightarrow})$ are left and right-grade membership data points respectively. This can be rewritten as

$$\vec{P} = \left\{\vec{P}_i = (x_i, y_i) \,|\, i = 0,1,..,n\right\} \tag{3}$$

for all i, $\vec{P}_i = \left\langle \vec{P}_i^{\leftarrow}, P_i, \vec{P}_i^{\rightarrow}\right\rangle$ with $\vec{P}_i^{\leftarrow}$, P_i and $\vec{P}_i^{\rightarrow}$ are left FDPs, crisp data point and right FDPs respectively. The formation process of FDP can be shown as in Fig. 2 and Fig. 3. (Zakaria, Wahab & Gobithaasan, 2015).

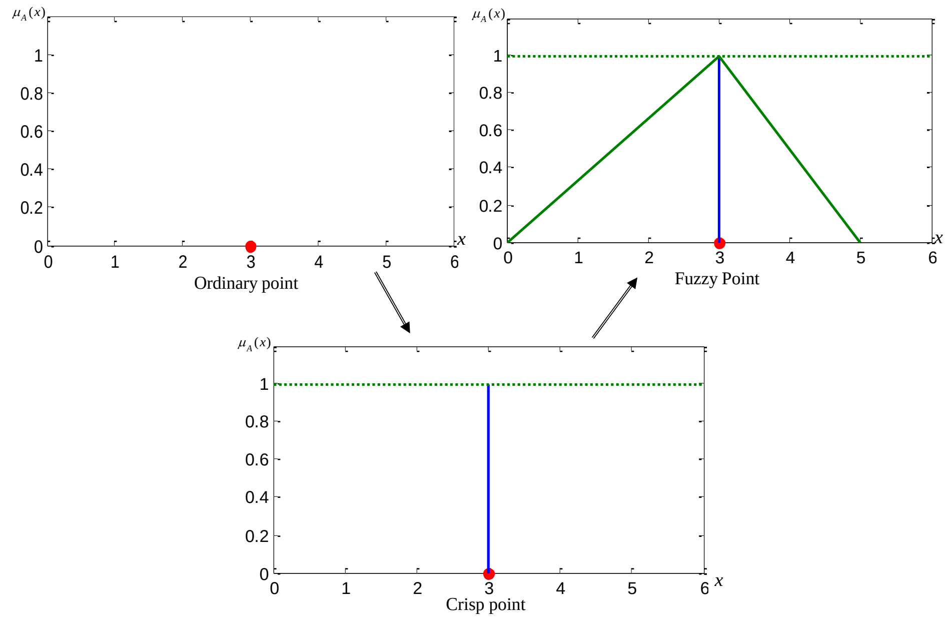

Figure 1: Process of defining FDP.

After FDPs had been defined, the next step is the alpha-cut operation (fuzzification process) which is applied to obtain new FDPs with a new fuzzy interval. The following definition elaborates the fuzzification process for FDPs.

Definition 7. Let $\overleftrightarrow{P}$ be the set of FDPs with $\overleftrightarrow{P}_i \in \overleftrightarrow{P}$. Then $\overleftrightarrow{P}_\alpha$ is the α-cut operation of FDPs which is given by

$$\overleftrightarrow{P}_{i_\alpha} = \left\langle \overleftrightarrow{P}^{\leftarrow}_{i_\alpha}, P_i, \overleftrightarrow{P}^{\rightarrow}_{i_\alpha} \right\rangle \tag{4}$$

where , P_i and $\overleftrightarrow{P}^{\rightarrow}_{i_\alpha}$ are α-cut of left FDPs, crisp data points and α-cut of right FDPs.

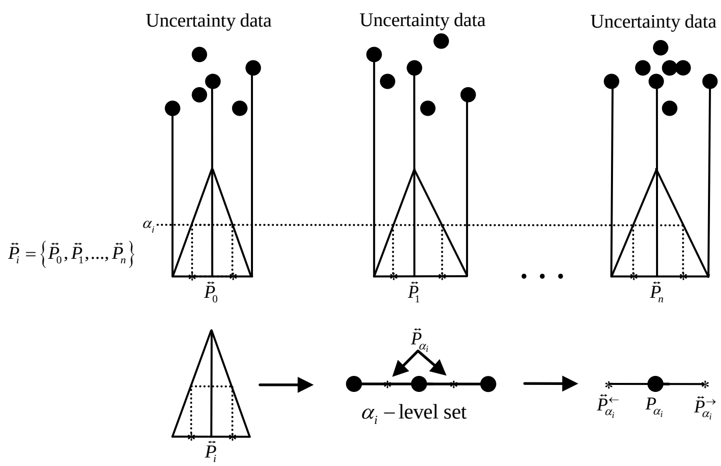

Figure 2. FDPs and the interval $\left\langle \overleftrightarrow{P}^{\leftarrow}_{\alpha_i}, P_{\alpha_i}, \overleftrightarrow{P}^{\rightarrow}_{\alpha_i} \right\rangle$ at α_i-level set in $(0,1]$.

Fig. 2 shows that the operation of α-cut towards FDPs which gives the interval $\left\langle \vec{P}^{\leftarrow}_{\alpha_i}, P_{\alpha_i}, \vec{P}^{\rightarrow}_{\alpha_i} \right\rangle$ at α_i-level set values in $(0,1]$. After carrying out α-cut operation against FDPs, we obtain the new FDP and new fuzzy interval which represent the crisp fuzzy solution and it is located within the fuzzy interval. The α-cut operation which was applied towards FDPs is not the final solution to obtain a crisp fuzzy solution. To obtain the final fuzzy solution, we need to defuzzify the α-FDPs to become a crisp fuzzy solution to represent the data point. The defuzzification method is elaborated in Def. 8.

Definition 8. Let α_i be the α-cut for every FDPs, $\vec{P}_i$ with $i = 0,1,\ldots,n$. Then $\bar{P}_i$ denoted defuzzification α-FDPs for $\vec{P}_i$ if for every $\vec{P}_i \in \vec{P}$,

$$\bar{P} = \left\{ \bar{P}_i \right\} \quad \text{for} \quad i = 0,1,\ldots,n \tag{5}$$

where for every $\bar{P}_i = \dfrac{\sum_{i=0} \left\langle \vec{P}^{\leftarrow}_i + P_i + \vec{P}^{\rightarrow}_i \right\rangle}{3}$ which $i = 0,1,\ldots,n$.

Using the fuzzy data points stated above, a fuzzy rational Bezier surface model can be expressed as follows

$$\vec{S}(u,v) = \frac{\sum_{i=0}^{n} \sum_{j=0}^{m} \vec{w}_{ij} B_i^n(u) \vec{P}_{ij} B_j^m(v)}{\sum_{k=0}^{n} \sum_{l=0}^{m} \vec{w}_{kl} B_k^n(u) B_l^m(v)} \qquad 0 \le u,v \le 1 \tag{6}$$

where $\vec{P}_{ij} = \left\langle \vec{P}^{\leftarrow}_{ij}, P_{ij}, \vec{P}^{\rightarrow}_{ij} \right\rangle$ are fuzzy control points which give fuzzy control net which $\vec{P}^{\leftarrow}_{ij}$ and $\vec{P}^{\rightarrow}_{ij}$ are left or lower and right or upper fuzzy control points and P_{ij} are crisp control points with $i = 0,1,\ldots,n$ and $j = 0,1,\ldots,m$.

When all the fuzzy weights $\vec{w}_{ij}$ are set to one, then the Eq. 6 reduces to a simpler form of fuzzy Bezier surface. The fuzzy weights serve as additional shape parameters instead of the parameters *u* and *v* are used to develop fuzzy surface. When $m = n = 3$ is substituted in Eq. 6, we obtain as fuzzy bicubic Bezier surface as illustrated in Fig. 3.

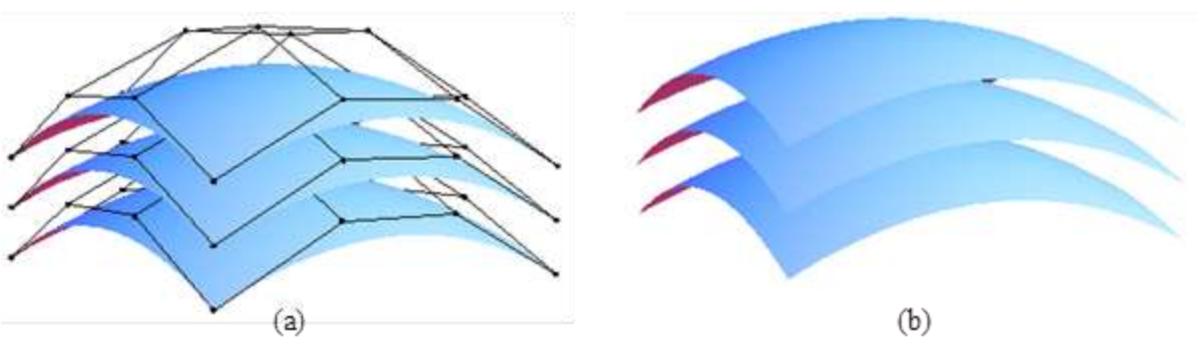

Figure 3. Uniform fuzzy rational bicubic Bezier surface consists 16 fuzzy data points (a) with fuzzy control net (b) without fuzzy control net.

A uniform fuzzy rational bicubic Bezier surface was constructed via Eq. 6 which interpolates fuzzy diagonal data points of fuzzy bicubic Bezier surface as shown in Fig. 3. The fuzzy surface has three layers due to the definition of fuzzy control points of fuzzy rational bicubic Bezier surface model in Eq. 6. The first and third layers are the upper and lower fuzzy rational bicubic Bezier surface, while the second layer is crisp rational bicubic Bezier surface. The fuzzy rational Bezier surface can be reshaped by modifying the values of fuzzy weights which is shown through Fig. 4.

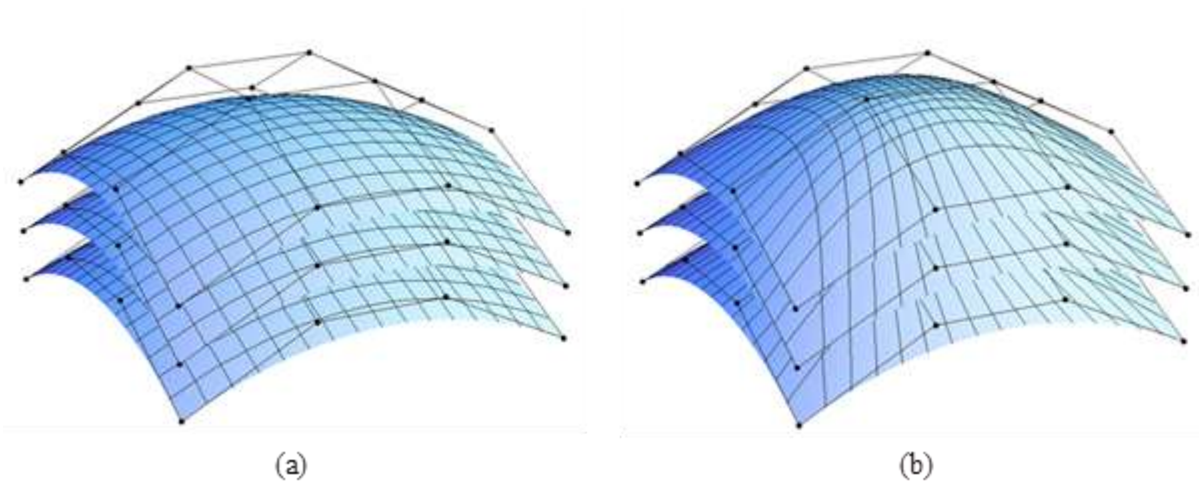

Figure 4. The change of fuzzy rational bicubic Bezier surface shape (a) before and (b) after changing the fuzzy weights.

RESULT AND DISCUSSION

This section discusses about the construction of FIRBBS model which is developed by using the definitions of fuzzy data and also the rational bicubic Bezier surface. The proposed FIRBBS model is constructed with piecewise fuzzy rational cubic Bezier function (Zakaria & Wahab 2014) which is the extension of fuzzy rational bicubic Bezier function $C(x,y)$ over a rectangular domain defined as $D=[x_0,x_m]\times[y_0,y_n]$. Let $\pi: a=x_0<x_1...<x_m=b$ be partition of $[a,b]$ and $\lambda: c=y_0<y_1<...<y_n=d$ be partition of $[c,d]$. The fuzzy rational bicubic Bezier function is defined over each rectangular patch $[x_i,x_{i+1}]\times[y_j,y_{j+1}]$, where $i=0,1,2,...,m-1$; $j=0,1,2,...,n-1$ as:

$$\overleftrightarrow{C}(x,y)=\overleftrightarrow{C}_{i,j}(x,y)=A_i(u)\vec{F}(i,j)\underline{A}_j^T(v), \tag{7}$$

where

$$\ddot{\vec{F}}(i,j)=\begin{pmatrix} \ddot{\vec{F}}_{i,j} & \ddot{\vec{F}}_{i,j+1} & \ddot{\vec{F}}^{y}_{i,j} & \ddot{\vec{F}}^{y}_{i,j+1} \\ \ddot{\vec{F}}_{i+1,j} & \ddot{\vec{F}}_{i+1,j+1} & \ddot{\vec{F}}^{y}_{i+1,j} & \ddot{\vec{F}}^{y}_{i+1,j+1} \\ \ddot{\vec{F}}^{x}_{i,j} & \ddot{\vec{F}}^{x}_{i,j+1} & \ddot{\vec{F}}^{xy}_{i,j} & \ddot{\vec{F}}^{xy}_{i,j+1} \\ \ddot{\vec{F}}^{x}_{i+1,j} & \ddot{\vec{F}}^{x}_{i+1,j+1} & \ddot{\vec{F}}^{xy}_{i+1,j} & \ddot{\vec{F}}^{xy}_{i+1,j+1} \end{pmatrix},$$

$$A_i(u)=[a_0(u) \quad a_1(u) \quad a_2(u) \quad a_3(u)] \qquad \underline{A}_j(v)=[\underline{a}_0(v) \quad \underline{a}_1(v) \quad \underline{a}_2(v) \quad \underline{a}_3(v)]$$

with

$$a_0(u)=\frac{(1-u)^3+3u\alpha_i(1-u)^2}{q_i(u)}, \qquad \underline{a}_0(v)=\frac{(1-v)^3+3v\hat{\alpha}_j(1-v)^2}{q_j(v)},$$

$$a_1(u)=\frac{u^3+3u^2\beta_i(1-u)}{q_i(u)}, \qquad \underline{a}_1(v)=\frac{v^3+3v^2\hat{\beta}_j(1-v)}{q_j(v)},$$

$$a_2(u)=\frac{u(1-u)^2}{q_i(u)}, \qquad \underline{a}_2(v)=\frac{v(1-v)^2}{q_j(v)},$$

$$a_3(u)=\frac{-u^2(1-u)}{q_i(u)}, \qquad \underline{a}_3(u)=\frac{-v^2(1-v)}{q_j(v)},$$

$$\ddot{q}_i(u)=(1-u)^3+3u\ddot{\alpha}_i(1-u)^2+3u^2\ddot{\beta}_i(1-u)+u^3$$

$$\ddot{q}_j(v)=(1-v)^3+3v\ddot{\hat{\alpha}}_j(1-v)^2+3v^2\ddot{\hat{\beta}}_j(1-v)+v^3.$$

Substituting A, $\ddot{\vec{F}}$ and $\underline{A}$ in Eq. 7, the fuzzy rational bicubic Bezier function $\ddot{C}(x,y)$ can be expressed as:

$$\ddot{C}(x,y)=\frac{(1-u)^3\ddot{\sigma}_{i,j}+3u(1-u)^2\ddot{\tau}_{i,j}+3u^2(1-u)\ddot{\varepsilon}_{i,j}+u^3\ddot{\kappa}_{i,j}}{(1-u)^3+3u\ddot{\alpha}_i(1-u)^2+3u^2\ddot{\beta}_i(1-u)+u^3} \tag{8}$$

where

$$\vec{\sigma}_{i,j} = \left[(1-v)^3 \vec{F}_{i,j} + 3v(1-v)^2 (\hat{\vec{\alpha}}_j \vec{F}_{i,j} + \vec{F}^y_{i,j}) + 3v^2(1-v)(\hat{\vec{\beta}}_j F_{i,j+1} - \vec{F}^y_{i,j+1}) + v^3 \vec{F}_{i,j+1} \right] \Big/ \vec{q}_j(v),$$

$$\vec{\tau}_{i,j} = \Big[(1-v)^3 (\vec{\alpha}_i \vec{F}_{i,j} + \vec{F}^x_{i,j}) + 3v(1-v)^2 \left(\hat{\vec{\alpha}}_j (\vec{\alpha}_i \vec{F}_{i,j} + \vec{F}^x_{i,j}) + \vec{\alpha}_i \vec{F}^y_{i,j} + \vec{F}^{xy}_{i,j} \right)$$

$$+3v^2(1-v)\left(\hat{\vec{\beta}}_j (\vec{\alpha}_i \vec{F}_{i,j+1} + \vec{F}^x_{i,j+1}) - \vec{\alpha}_i \vec{F}^y_{i,j+1} - \vec{F}^{xy}_{i,j+1} \right) + v^3 (\vec{\alpha}_i \vec{F}_{i,j+1} + \vec{F}^x_{i,j+1}) \Big] \Big/ \vec{q}_j(v),$$

$$\vec{\varepsilon}_{i,j} = \Big[(1-v)^3 (\vec{\beta}_i \vec{F}_{i+1,j} - \vec{F}^x_{i+1,j}) + 3v(1-v)^2 \left(\hat{\vec{\alpha}}_j (\vec{\beta}_i \vec{F}_{i+1,j} - \vec{F}^x_{i+1,j}) + \vec{\beta}_i \vec{F}^y_{i+1,j} - \vec{F}^{xy}_{i+1,j} \right)$$

$$+3v^2(1-v)\left(\hat{\vec{\beta}}_j (\vec{\beta}_i \vec{F}_{i+1,j+1} - \vec{F}^x_{i+1,j+1}) - \vec{\beta}_i \vec{F}^y_{i+1,j+1} + \vec{F}^{xy}_{i+1,j+1} \right) + v^3 (\vec{\beta}_i \vec{F}_{i+1,j+1} + \vec{F}^x_{i+1,j+1}) \Big] \Big/ \vec{q}_j(v),$$

$$\vec{\kappa}_{i,j} = \Big[(1-v)^3 \vec{F}_{i+1,j} + 3v(1-v)^2 (\hat{\vec{\alpha}}_j \vec{F}_{i+1,j} + \vec{F}^y_{i+1,j}) + 3v^2(1-v)(\hat{\vec{\beta}}_j \vec{F}_{i+1,j+1} - \vec{F}^y_{i+1,j+1})$$

$$+v^3 \vec{F}_{i+1,j+1} \Big] \Big/ \vec{q}_j(v).$$

Since the free fuzzy parameter such as $\vec{\alpha}_i, \vec{\beta}_i, \hat{\vec{\alpha}}_j$ and $\hat{\vec{\beta}}_j$ correspond to the entire network of fuzzy curve, thus there is no local control over the fuzzy surface. Therefore, the variables of fuzzy weights Since the free fuzzy parameter such as $\vec{\alpha}_i, \vec{\beta}_i, \hat{\vec{\alpha}}_j$ and $\hat{\vec{\beta}}_j$ correspond to the entire network of fuzzy curve, thus there is no local control over the fuzzy surface. Therefore, the variables of fuzzy weights are introduced to overcome this problem in which the desired local can be achieved. The new free fuzzy parameters $\vec{\alpha}_i, \vec{\beta}_i, \hat{\vec{\alpha}}_j$ and $\hat{\vec{\beta}}_j$ are now given as,

$$\vec{\alpha}_i(y_j) = \vec{\alpha}_{i,j},\ \vec{\beta}_i(y_j) = \vec{\beta}_{i,j},\ \hat{\vec{\alpha}}_j(x_i) = \hat{\vec{\alpha}}_{i,j},\ \hat{\vec{\beta}}_j(x_i) = \hat{\vec{\beta}}_{i,j}$$
$$i = 0,1,2,\ldots,m-1; \quad j = 0,1,2,\ldots,n-1.$$

The shape of the fuzzy surface can be modified by assigning different values to these fuzzy parameters. The fuzzy weights impose different constraint on $\vec{\alpha}_i, \vec{\beta}_i, \hat{\vec{\alpha}}_j$ and $\hat{\vec{\beta}}_j$.

The parameters of $\vec{F}^x_{i,j}, \vec{F}^y_{i,j}$ and $\vec{F}^{xy}_{i,j}$ are determined arithmetic mean method were mentioned in (Hussain & Hussain 2006b, 2007, 2006a). Since Eq. 7 which involves the fuzzy function, hence the arithmetic mean method need to be converted into the fuzzy arithmetic mean method.

This fuzzy arithmetic mean method is the three-point difference approximation based on arithmetic manipulation. This method is defined as:

$$\vec{F}^x_{0,j} = \vec{\Delta}_{0,j} + (\vec{\Delta}_{0,j} - \vec{\Delta}_{1,j})$$

$$\vec{F}^x_{i,j} = \frac{\vec{\Delta}_{i,j} + \vec{\Delta}_{i-1,j}}{2}, \qquad i = 1,2,3,\ldots,m-1; \quad j = 0,1,2,\ldots,n,$$

$$\vec{F}^x_{m,j} = \vec{\Delta}_{m-1,j} + (\vec{\Delta}_{m-1,j} - \vec{\Delta}_{m-2,j}).$$

$$\overleftrightarrow{F}^{y}_{i,0} = \underline{\overleftrightarrow{\Delta}}_{i,0} + (\underline{\overleftrightarrow{\Delta}}_{i,0} - \underline{\overleftrightarrow{\Delta}}_{i,1}),$$

$$\overleftrightarrow{F}^{y}_{i,j} = \frac{\underline{\overleftrightarrow{\Delta}}_{i,j} + \underline{\overleftrightarrow{\Delta}}_{i,j-1}}{2}, \qquad i = 0,1,2,\dots,m; \;\; j = 1,2,3,\dots,n-1,$$

$$\overleftrightarrow{F}^{y}_{i,n} = \underline{\overleftrightarrow{\Delta}}_{i,n-1} + (\underline{\overleftrightarrow{\Delta}}_{i,n-1} - \underline{\overleftrightarrow{\Delta}}_{i,n-2}).$$

$$\overleftrightarrow{F}^{xy}_{i,j} = \frac{(\overleftrightarrow{F}^{x}_{i,j+1} - \overleftrightarrow{F}^{x}_{i,j-1}) + (\overleftrightarrow{F}^{y}_{i+1,j} - \overleftrightarrow{F}^{y}_{i-1,j})}{2}, \qquad i = 1,2,\dots,m-1; \;\; j = 1,2,\dots,n-1,$$

where $\overleftrightarrow{\Delta}_{i,j} = \overleftrightarrow{F}_{i+1,j} - \overleftrightarrow{F}_{i,j}$ and $\underline{\overleftrightarrow{\Delta}}_{i,j} = \overleftrightarrow{F}_{i,j+1} - \overleftrightarrow{F}_{i,j}$. Thus, Fig. 5 illustrates FIRBBS model by using Eq. 7 which using the FDPs.

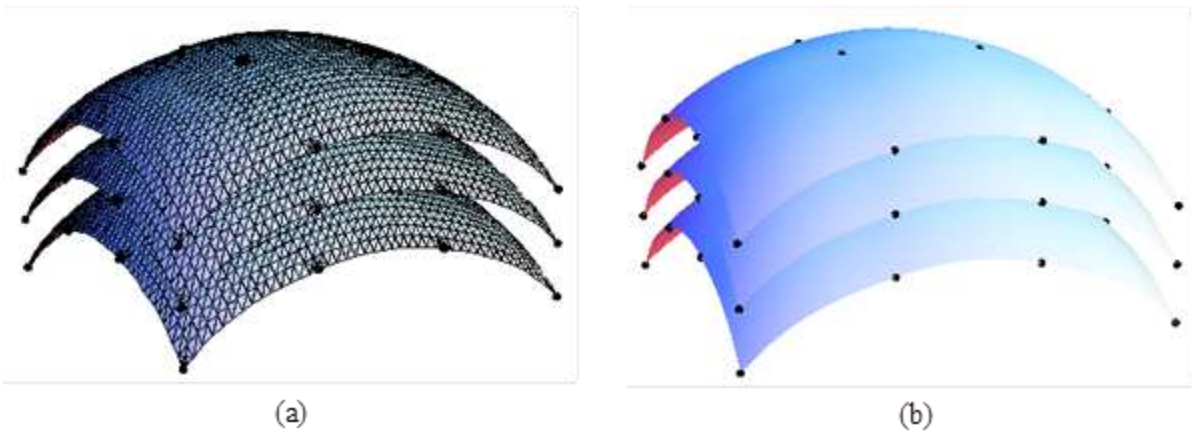

Figure 5. Uniform FIRBBS (a) with meshes (b) without meshes which interpolates given FDPs.

In Fig. 5a and Fig. 5b, FIRBBS model constructed using Eq. 7 which interpolates all 16 uniform FDPs. The fuzzy weight can be assigned with different values to change the shape of the fuzzy surface as illustrated in Fig. 6.

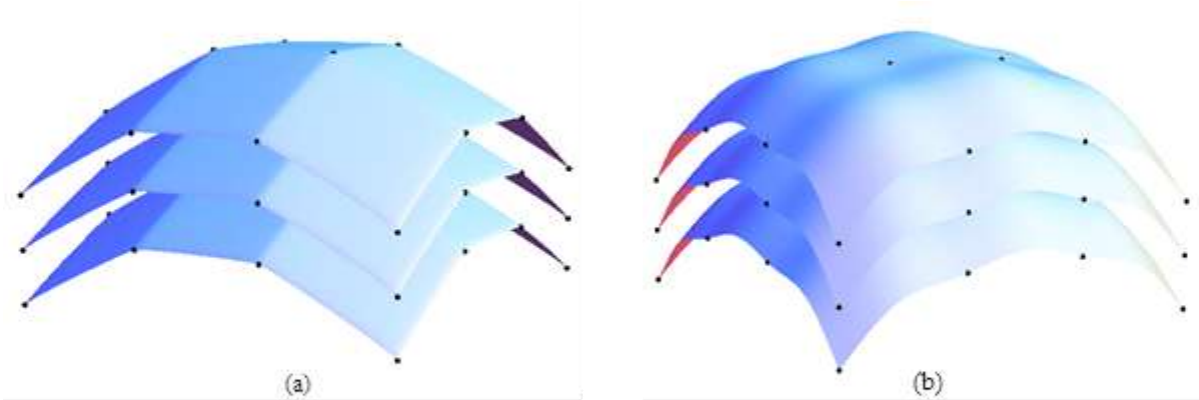

Figure 6. Uniform fuzzy weight changes with all fuzzy weights (a) equals to 6.5 and (b) equals to 0.5.

Fig. 6 shows the effect of fuzzy weights changes involved in designing various shape. These fuzzy values, $\vec{\alpha}_{i,j}$, $\vec{\beta}_{i,j}$, $\overleftrightarrow{\alpha}_{i,j}$ and $\overleftrightarrow{\beta}_{i,j}$ are set as $\vec{\alpha}_{i,j} = \vec{\beta}_{i,j} = \overleftrightarrow{\alpha}_{i,j} = \overleftrightarrow{\beta}_{i,j} = 6.5$ shows uniform fuzzy surface shape as in Fig. 10a and $\vec{\alpha}_{i,j} = \vec{\beta}_{i,j} = \overleftrightarrow{\alpha}_{i,j} = \overleftrightarrow{\beta}_{i,j} = 0.5$ shows the uniform fuzzy surface shape as in Fig. 6b.

After FIRBBS was defined successfully, then this FIRBBS is apply to model the number of fuzzy data of lakebed modeling by using the uncertainty bathymetry data. Therefore, the issues of lakebed modeling had been proposed by (Wahab 2008; Zakaria et al. 2014) due to some errors existed which used various surface function including B-spline surface function. Thus, the modeling process of FIRBBS until defuzzification process can be represented as Algorithm1 as follows.

Algorithm 1.

Step 1: Define the uncertain data via Def. 6.

Step 2: Integrate the fuzzy data together with FIRBBS model which is resulted based on Eq. 7 and Fig. 5.

Step 3: Apply the alpha-cut operation as the fuzzification process which was mentioned in (Zakaria et al. 2014).

Step 4: Defuzzify the fuzzified FIRBBS model followed by (Zakaria et al. 2014).

Based on the Algorithm 1, then we applied the scientific data as an example of the using FIRBBS model in real data modeling. We choose the seabed data modeling by selected sixteen data points which can be illustrated through Figure 7a until Figure 7c.

To identify the effectiveness of this FIRBBS model, the error for every data and percentage error are calculated. If the error values are small, then this model is acceptable in modeling those data points. To calculates these error, we need to use the error equation as mentioned in (Zakaria et al. 2014). The error can be plotted and shows in Fig. 7d. The percentage error of this modeling is 0.0371648 which is accepted for this modeling.

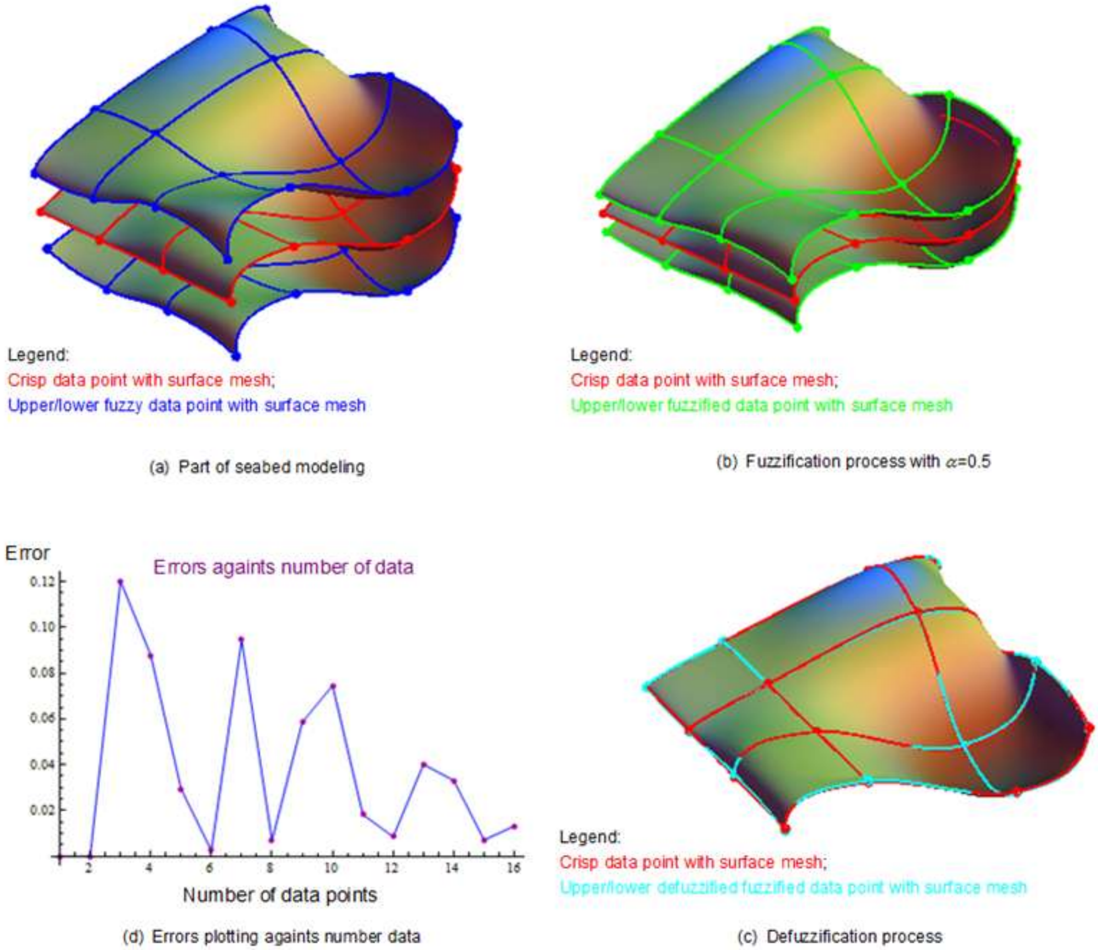

Figure 7. The FIRBBS modeling with (a) part of seabed data, (b) fuzzified, (c) defuzzified fuzzified models and (d) errors plotting.

CONCLUSION

Fuzzy interpolation surface model which represented by FIRBBS model shows that the applicable on using the fuzzy set theory and interpolation Bezier surface function in defining and modeling the uncertainty data points. This purposed model also can be used to model the part of seabed data which can be shown in previous figures. Based on the seabed modeling through FIRBBS model, this modeling was successful according to the errors and percentage error were obtained which the percentage error is smaller and acceptable for seabed modeling. Therefore, the FIRBBS model can be used in modeling the uncertainty data of surface model.

This FIRBBS model can be extended for modeling the large data points which has uncertain properties. Therefore, this model can be used as piecewise FIRBBS model which fulfilled the continuity condition at least C^1 continuity. Besides that, this model can be extended to rational B-spline or non-uniform rational B-spline (NURBS) function as a further research which give more controlling of obtaining surface.

ACKNOWLEDGEMENT

The authors would like to acknowledge Universiti Malaysia Sabah and Ministry of Higher Education (MOHE), Malaysia, for providing the facilities to conduct this research. They further acknowledge anonymous referees for their constructive comments which improved the readability of this paper.

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