

ANALYSIS OF VIBRATOR FOR SOMPOTON USING CANTILEVER BEAM MODEL

Tee Hao Wong¹, M. C. D. Ngu¹, Jackson H. W. Chang¹, Jedol Dayou¹ & W. Y. H. Liew²

¹Energy, Sound and Vibration Research Group (e-VIBS),
School of Science & Technology,
Universiti Malaysia Sabah, Jalan UMS, 88400 Kota Kinabalu, Sabah, Malaysia.

²School of Engineering & Information Technology,
Universiti Malaysia Sabah, Jalan UMS, 88400 Kota Kinabalu, Sabah, Malaysia.

ABSTRACT. *One of the famous heritages in Sabah is the traditional musical instrument called sompoton. This instrument has several parts with the vibrator being the most important of all. In this paper, the vibrator is modeled as a cantilever beam with uniformly distributed mass. Using this model, the fundamental frequency is analyzed using Rayleigh's energy theory. The vibrator made from aluminum is fabricated at different dimensions and is excited using constant air jet to obtain its fundamental resonance frequency. The measured fundamental frequency is then compared with the theoretical values calculated based on cantilever beam model and Rayleigh's energy theories. It was found that the experimental and theoretical results exhibit the same trend but differ in magnitude. To overcome this, a correction factor is added to the theoretical formulation to account for fabrication error.*

KEYWORDS. Vibrator, cantilever beam, Rayleigh's energy theory, resonance frequency, single degree of freedom system.

INTRODUCTION

It is important to preserve the cultural heritage and at the same time move towards a developed country. Sabah is the second largest state in Malaysia and is richly blessed with nature diversity and unique heritages. It is renowned for its cultural diverse indigenous communities of more than 30 ethnic groups. The largest ethnic group in Sabah is Kadazan Dusun. Sompoton is undoubtedly one of their famous cultural heritages which is also an attraction in tourism. This traditional musical instrument consists of three parts- acoustic chamber, vibrator locally known as sodi and bamboo pipes (Figure 1) (Marasan, 2003).

In the past, only a few studies have scrutinized the sound production mechanism of musical instrument and even fewer studies dealt with traditional musical instruments. Someya & Okamoto (2007) studied the measurement of the flow and vibration of the Japanese traditional bamboo flute using the dynamic particle image velocimetry (PIV). They successfully visualized the air oscillation in the bamboo flute which is useful to understand the important phenomena in sound production of the instrument. Rujinirum *et al.* (2005) characterized the acoustics properties for different type of woods used to make Ranad (Thai traditional xylophone) and the resonator box. They managed to determine the dominant acoustic properties of the wood required to make a good quality of Ranad bars and resonator box. A wood with high specific dynamic Young's modulus, density and hardness is needed for Ranad box. As for the resonator box, a high value of acoustic converting efficiency (ACE) is necessary.

Far in the west in Finland, Erkut *et al.* (2002) studied the analysis of the sound generated from a Finnish traditional musical instrument-kantele, based on measurements and analytical formulation. A synthesis model has been proposed to capture the nonlinear

properties of kantele tones. The sounds produced were proven in accordance with the measurements and analytical approximations.

In the west part of Malaysia, Ismail *et al.* (2006) have studied the properties and characteristics of sound produced by kompang, Malay traditional musical instrument, and analyzed it using computer music synthesis. Kompang is noted as a pitch less musical instrument and it is similar to other vibrating circular membrane instruments. In Sabah (east Malaysia), Ong & Dayou (2009) initiated the study of frequency analysis of sound from local traditional musical instrument-sompoton. They reported that the generation of harmonic frequency from sompoton follows the open-end pipe model but the fundamental frequency does not comply with the same model.

Even until to date, very limited studies investigate the vibrator of sompoton. In view of this, it is important to initiate the studies in the effort to preserve and improvise this musical instrument. Vibrator plays an important role in sound production of sompoton. The original vibrator is made by polod which is a kind of palm tree found locally. Existing vibrator does not have a fixed standard dimensions to produce certain sound frequency. It depends on the expertise of the master. In this research, vibrator is constructed using aluminum instead of polod to facilitate a less complicated way to understand the sound production mechanism of sompoton. To carry out the work, vibrators at different dimensions were fabricated and analyzed in order to determine the governing formulation of the fundamental frequency.

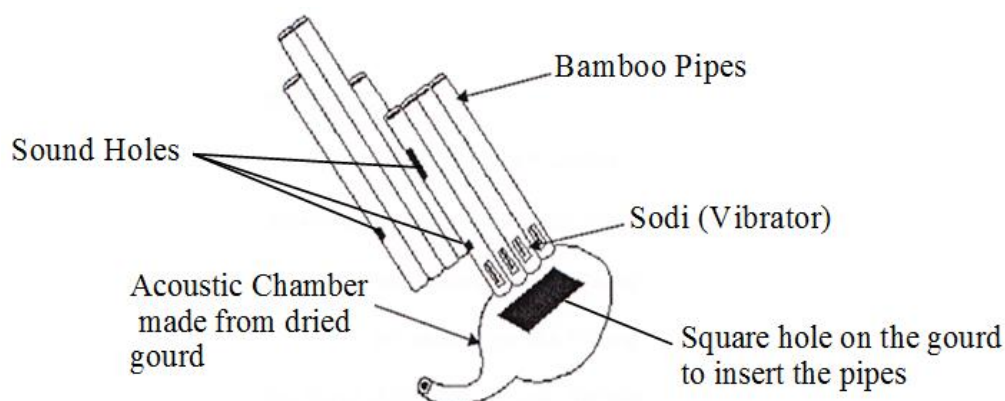
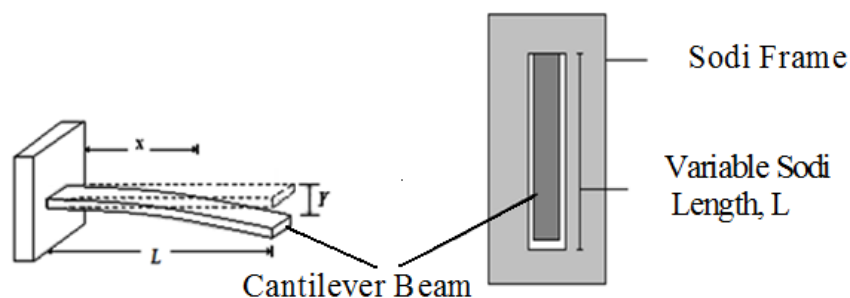


Figure 1. Structure design of Sompoton.

CANTILEVER BEAM MODEL AND FUNDAMENTAL FREQUENCY ANALYSIS OF SODI

Cantilever is a beam supported at one end and the other end can vibrate freely (Figure 2a). It is widely found in construction designs such as cantilever bridges, balconies and also applied on the aircraft wings design. Detailed inspection of the sompoton's vibrator in Figure 2b shows that generally it has a similar design as cantilever beam (Figure 2a) where one end of the vibrator is attached to the frame and the other end vibrates freely when it is subjected to force.



(a) Cantilever beam model (b) Schematic drawing of sodi
Figure 2. Schematic diagram of cantilever beam and vibrator (sodi).

Although a cantilever beam has many modes of vibration, the knowledge of its fundamental mode is prime important as the higher frequencies are a multiplication of this fundamental frequency known as harmonics. Therefore, in this paper, a single-degree-of-freedom system is adopted. The vibrator is modeled as a single mode cantilever beam to predict its fundamental resonance frequency.

The sompton’s vibrator has a uniform dimension and thus it is modeled as a cantilever beam with uniformly distributed mass as shown in Figure 3. When a uniformly distributed force is exerted on it, Rayleigh energy theorem can be used to determine its fundamental sound frequency. The angular frequency of a vibrating system can be written as (Serway & Jewett, 2006).

$$\omega = \sqrt{\frac{k}{M}} \quad (1)$$

where ω is the angular frequency in radian per second, k the stiffness of beam and M the mass.

The angular frequency can be expressed in the form of

$$\omega = 2\pi f. \quad (2)$$

Combining Equation (1) and (2) gives

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{M}} \quad (3)$$

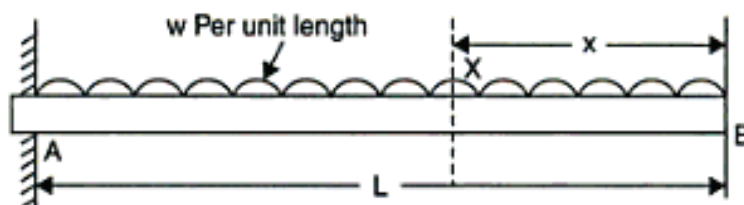


Figure 3. Cantilever beam of length L fixed at A and carrying a uniformly distributed mass of w per unit length over the entire length of the cantilever (Bansal, 2010).

The maximum deflection of the cantilever beam, y is given by (Gere & Goodno, 2008; Merriman, 1924).

$$y = \frac{wL^4}{8EI} \quad (4)$$

where E is the modulus of elasticity, I is the second moment of inertia, L is the length, and w is the uniform weight per unit length. Meanwhile, the stiffness of the beam system can be written as

$$k = \frac{F}{y} = \frac{wL}{y} \quad (5)$$

where F is the total force acting on the beam. Substitute Equation (4) into Equation (5) gives

$$k = \frac{8EI}{L^3}. \quad (6)$$

According to the Rayleigh energy theorem, the cantilever beam system is the same as the system of spring mass, so the stiffness k in Equation (3) can be substituted from Equation (6) to give the fundamental natural frequency of uniformly distribute mass as

$$f = \frac{1}{2\pi} \sqrt{\frac{8EI}{ML^3}}. \quad (7)$$

It can be seen that the fundamental frequency is directly proportional to modulus of elasticity, E and second moment of inertia, I and inversely proportional to mass, M and length of beam, L .

EXPERIMENTAL SETUP, RESULTS AND DISCUSSIONS

The aim of this paper is to establish a theoretical model that can explain the sound production mechanisms of sompoton. In the previous section, the hypothesis suggested that the sompoton's vibrator is similar to a cantilever beam due to the similarity of their structural designs. In this section, experimental works were performed to verify this hypothesis and also to investigate the governing factors that may affect the sound produced by the vibrator. To do this, vibrators at different lengths range from 17 mm to 25 mm were fabricated from aluminum material using Computer Numerical Control (CNC) machine to ensure uniformity. The range is chosen with the purpose of imitating the real sompoton's vibrator which usually ranges from 15 mm to 25 mm. The width and thickness of fabricated vibrator is fixed at 2 mm and 0.2 mm, respectively. These dimensions were appropriate in getting better audible sound from the vibrator. Note that it is hard to produce audible sound with longer width and thickness.

Figure 4 shows the experimental setup in this work. The vibrator was excited by constant air jet pressure from an air compressor. The beam of the vibrator acts as an air gate that alternately blocks and un-blocks the passing air. This generates a vibration in the surrounding air and thus producing audible tone as described by Hopkin (1996). The sound generated by the excitation of the vibrator is recorded using the Harmonie measurement system and later analysed using MATLAB to obtain the frequency spectrum. To avoid unwanted noise, the experiments were carried out in a noise free anechoic room. The aluminum vibrator has modulus of elasticity $E= 70$ GPa, second moment of inertia $I = Dt^3/12=1.333 \times 10^{-15}$ N and material density of $\rho = 2700 \text{ kg/m}^3$. Here, D denotes the width of the beam and t denotes the thickness of the beam.

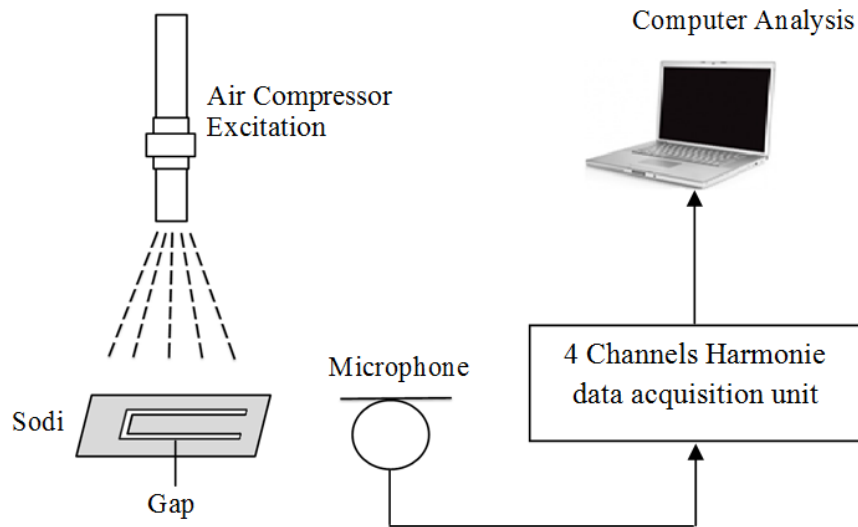


Figure 4. Experimental setup of Sompoton's vibrator excitation and data analyzing.

Table 1 shows the frequency value obtained from the experimental measurements for all vibrator lengths and its corresponding theoretical frequency calculated using Equation (7). It is clearly seen that both frequencies differ in magnitude. This is further visualized in the graphical comparison shown in Figure 5 where the experimental frequency is always higher than the theoretical value. Detailed inspection of Figure 5 shows that although they differ in magnitude, both data shared an identical trend that is the sound frequency decreases as the vibrator length increases in similar proportion. It is postulated that error incurred during the fabrication is a possible reason to explain the result differences. During the cutting process, the CNC machine's cutter defects the vibrator's beam and leaves a little curvature shape on the beam. As a result, each vibrator experiences similar fabrication defects in the same proportion.

Table 1. Experimental and theoretical value of fundamental frequency of the sound from aluminum vibrator.

Vibrator Length, mm	Experimental frequency f_e , Hz	Theoretical frequency f_t , Hz	Percentage difference Δ , %
17	572.00	457.90	19.95
18	567.00	408.44	27.96
19	520.00	366.58	29.50
20	505.70	330.84	34.58
21	413.00	300.08	27.34
22	392.40	273.42	30.32
23	359.00	250.16	30.32
24	338.00	229.75	32.03
25	304.10	211.73	30.37

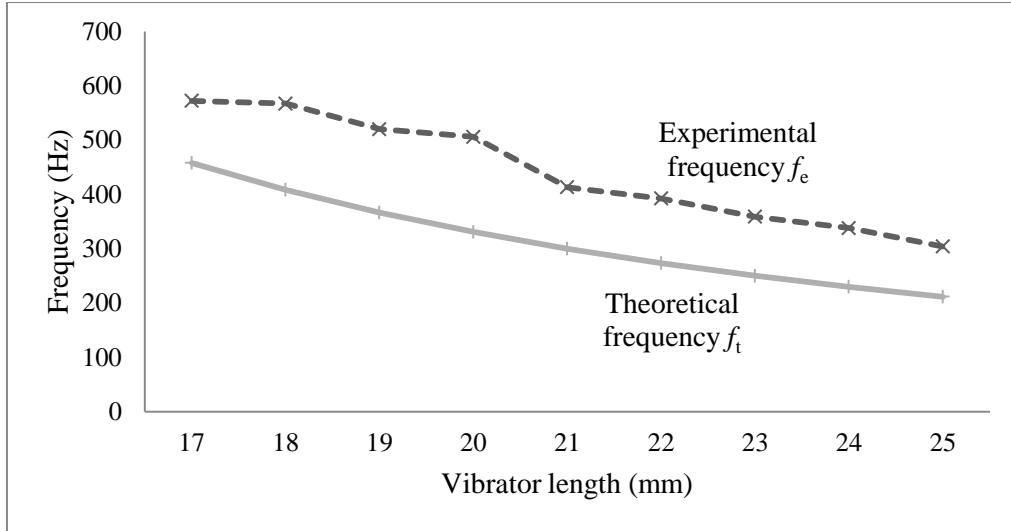


Figure 5. Comparison between the theoretical and experimental value of the fundamental frequency of aluminum vibrator with different length.

To account for the fabrication errors, a correction factor is added into the theoretical formula to determine the actual frequency as follows. The linear scale graph in Figure 5 is first rescaled into semi log of y-axis. In this way, the data obtained from experimental work ($\log f_e$) and also from theoretical prediction ($\log f_t$) are linearized in the parallel proportion as shown in Figure 6. For clarity, both equations are written in the following.

$$\log f_e = -0.036x + 2.817, \quad (8)$$

$$\log f_t = -0.041x + 2.692. \quad (9)$$

Substituting Equation (8) into (9) for x gives, after rearrangement as

$$0.041 \log f_e - 0.036 \log f_t = 0.018585. \quad (10)$$

This can also be rewritten as

$$\log \left(\frac{f_e^{0.041}}{f_t^{0.036}} \right) = 0.018585. \quad (11)$$

From the log identity, it can be written that

$$\frac{f_e^{0.041}}{f_t^{0.036}} = 1.04372239 \quad (12)$$

or

$$f_e^{0.041} = 1.04372239 f_t^{0.036}. \quad (13)$$

Rearranging the equation gives

$$f_e = \sqrt[0.04]{1.04372239 f_t^{0.036}}, \quad (14)$$

which gives the final equation as

$$f_e = 2.84 f_t^{0.88} \quad (15)$$

rounded to two decimal points where f_t is the theoretical value of frequency of the fabricated vibrator given in Equation (7).

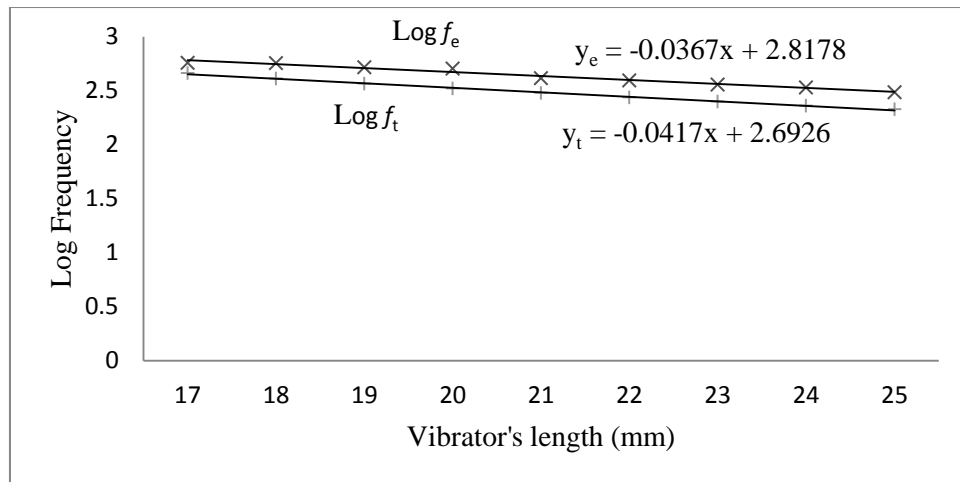


Figure 6. Conversion of graph into logarithm form.

Equation (15) is the relationship between the experimental and theoretical value of the vibrator’s frequency. It is the corresponding equation that gives the actual frequency in terms of theoretical equation with correction factor. This means that using the same CNC machine setting, the required thickness of the vibrator has to be adjusted according to this equation and not to the theoretical formulation in Equation (7).

In order to validate this new formula, the theoretical value of the fabricated vibrator frequency in Table 1 was substituted into Equation (15) and then compared with the actual measurement. Table 2 shows the comparison between the two values. Detailed inspection of the table shows that the two set of values are in close agreement with a maximum deviation of 8.43%. It can also be seen in Figure 7 that the corrected frequency closely matches the experimental frequency. Both graphs show a similar decreasing trend as the vibrator’s length decreases.

Table 2. Experimental and corrected formula result data of aluminum vibrator length test.

Vibrator Length, mm	Experimental frequency f_e , Hz	Corrected formula frequency f , Hz	Percentage difference Δ , %
17	572.00	615.99	7.69
18	567.00	557.17	1.73
19	520.00	506.70	2.56
20	505.70	463.06	8.43
21	413.00	425.03	2.91
22	392.40	391.69	0.18
23	359.00	362.28	0.91
24	338.00	336.19	0.53
25	304.10	312.92	2.90

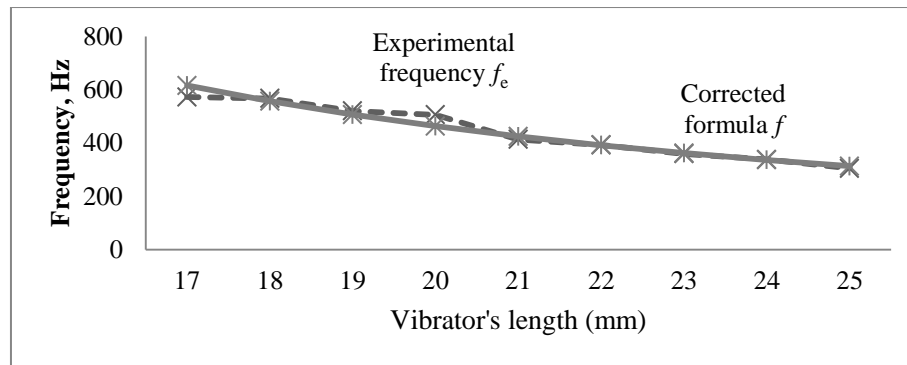


Figure 7. Comparison graph of experimental and corrected theoretical formula.

CONCLUSIONS

This paper presented the modeling of aluminum vibrator of uniformly distributed mass cantilever beam model using Rayleigh's energy theories. Experiments had been carried out to measure the frequency produced by vibrator specimens of different lengths. The results were compared with the theoretical formula. The frequency obtained in the experiment using vibrator at different lengths exhibit similar pattern but differ in magnitude when compared to theoretical value. To solve the problem, new corrected theoretical formula was derived. Frequency was found to be inversely proportional to the vibrator length as indicated in the model. This finding has a significant importance especially in the future research of the vibrator modelling and generally in the research of sompoton or other local traditional musical instruments.

REFERENCES

- Bansal, R. K. 2010. *A Text Book of Strength of Materials (4th ed)*. Daryaganj, New Delhi: Laxmi Publications (P) Ltd.
- Erkut, C., Karjalainen, M., Huang, P., & Välimäki, V. 2002. Acoustical Analysis and Model-Based Sound Synthesis of the Kantele. *Journal of the Acoustical Society of America*, **112**(4): 1681-1691.
- Gere, J. M., & Goodno, B. J. 2008. *Mechanics of Materials (7th ed)*. Toronto, Kanada: Cengage Learning.
- Hopkin, B. 1996. *Musical Instrument Design*. Tucson, Arizona: See Sharp Press.
- Ismail, A., Samad, S. A., Hussain, A., Azhari, C. H., & Zainal, M. R. M. 2006. *Analysis of the Sound of the Kompang for Computer Music Synthesis*. IEEE 4th Student Conference on Research and Development (SCoReD 2006). 27-28 June 2006, Shah Alam, Selangor, Malaysia.
- Marasan, R. 2003. *Alat Muzik Sompoton Negeri Sabah*. Pejabat Kebudayaan dan Kesenian Negeri Sabah, Sabah, Malaysia.
- Merriman, M. 1924. *Mechanics of Materials*. John Wiley & Sons.
- Ong, C. W., & Dayou, J. 2009. Frequency Characteristic of Sound from Sompoton Musical Instrument. *Borneo Science*, **25**: 71-79.
- Rujinirum, C., Phinyocheep, P., Prachyabrued, W., & Laemsak, N. 2005. Chemical Treatment of Wood for Musical Instruments. Part I: Acoustically Important Properties of Wood for the Ranad (Thai Traditional Xylophone). *Wood Science and Technology*, **39**(1): 77-85.

- Serway, R. A., & Jewett, J. W. 2006. *Principles of Physics: A Calculus-Based Text (4th ed.)*, USA: Brooks/Cole-Thomson Learning.
- Someya, S., & Okamoto, K. 2007. Measurement of the Flow and Its Vibration in Japanese Traditional Bamboo Flute Using the Dynamic PIV. *Journal of Visualization*, **10(4)**: 397-404.