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# IMAGE COMPRESSION BY DISCRETE COSINE TRANSFORMATION

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***ABSTRACT:*** *The discrete cosine transform (DCT) is now well recognized as one of the most important technique in image data compression. Among the class of transforms possessing fast computational algorithms, the cosine transform has a superior energy compaction property. Due to its simple implementation scheme, the DCT is widely used as a substitute to the optimal Karhunen Loeve (KL) transform.*
*In this paper, the discrete cosine transform is presented and an algorithm for its implementation is developed. The picture is firstly transform coded using 8x8 sub-blocks then a quantization and an entropy coding are used.*

**Key words:** DCT transform, data compression, quantization, coding.

## INTRODUCTION

The digital techniques for image coding are having a very fast increasing development in this data transmission era (Digital transmission for all kinds of utilization).
Large amounts of memory space as well as wide channel are needed to save or transmit an image. In fact these are the most important problems facing the information technology today. Therefore it is imperative to reduce the number of necessary bits to encode an image. This requires an appropriate model for image compression based on the characteristics and the statistical properties of the image.

The orthogonal transformations (Fourier, Haar, Hadamard, KL, Cosine…) are well adapted for this kind of compression (WINTZ 1972). They use the important correlation properties that exist on a visual support (continuity, correlation of the samples in an image…) by eliminating the unnecessary redundant information. The principle of these methods is to operate on the image a change of the basis in such a way that the samples in the new basis are uncorrelated.

In this paper, a brief comparison of the orthogonal transforms is presented. The DCT and its effect on the process of compression are then discussed.

## METHODOLOGY

The image transformations refer to a class of unitary matrices used to represent images. Most of the unitary transforms tend to concentrate a great amount of the image energy on only few transform coefficients. Consider a first order stationary Markov sequence of length N. The distortion expressed by the MSE decreases as the rate expressed in bits per sample increases, for all the transforms; but the best performance is obtained with the cosine and the KL transforms (ROSENFIELD 1982).

The KL Transform is optimal in the sense of MSE versus bit rate, however it requires a large amount of computation. In practice other fast transforms are been used such as the cosine, the sine, Fourier, Hadamard (ROSENFIELD 1982).

Figure 1 compares the performances of the different transforms. We notice that the cosine transform outperforms all the other transforms and performs as well as the KL.

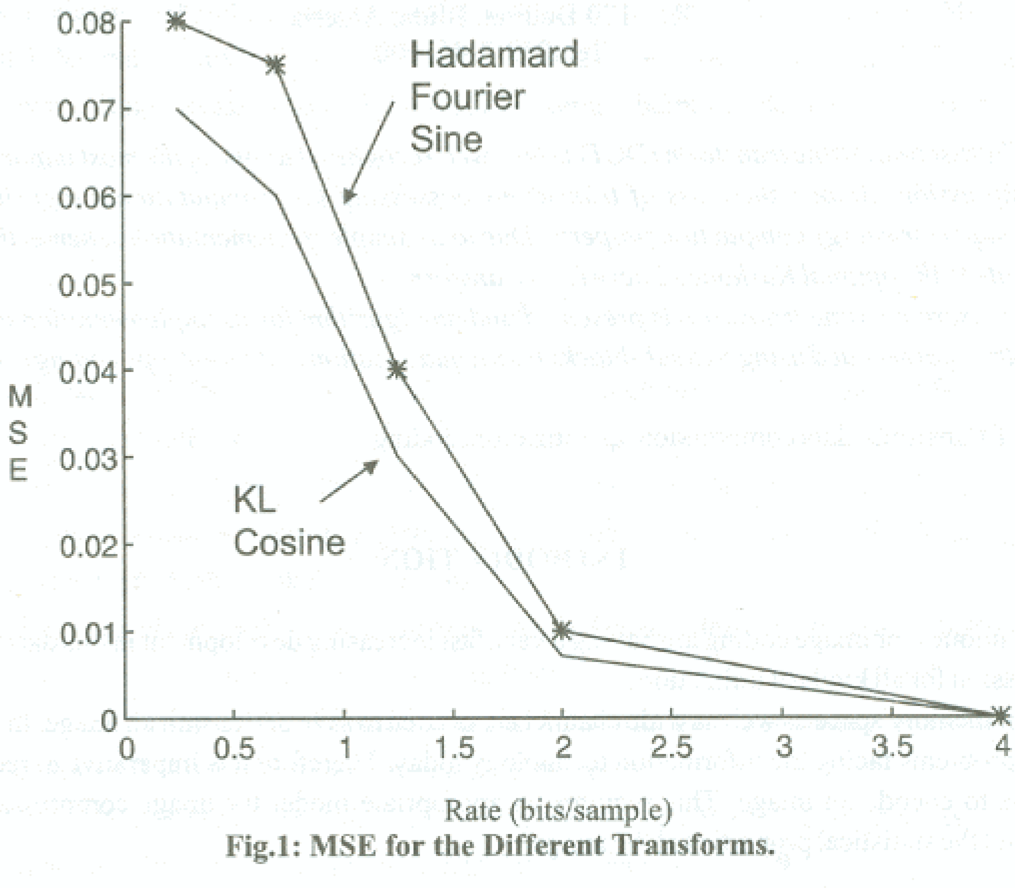



**Fig.1: MSE for the Different Transforms.**

The compression algorithm described in this work uses the DCT and is divided in three phases (HOU 1987) as in Fig.2.

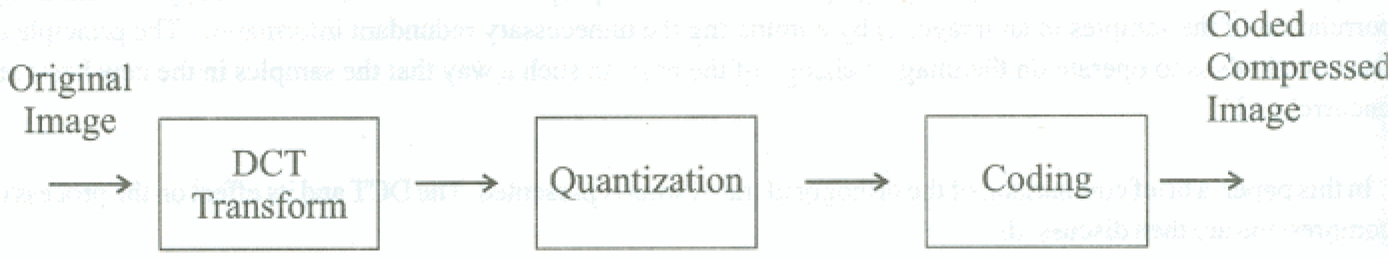



**Fig. 2 : Compression Scheme.**

## THE DISCRETE COSINE TRANSFORM

The key of the compression process described here is the DCT. This transform converts a spatial image into a spectral image. The original image can be recovered using the Inverse transform IDCT.

### Definition of the DCT

The DCT for a 2D signal of dimension NN is defined as follows:

$$DCT(i,j) = \frac{C(i)C(j)}{\sqrt{2N}} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} pixel(x,y) \cos\left[\frac{(2x+1)i\pi}{2N}\right] \cos\left[\frac{(2y+1)j\pi}{2N}\right]$$

Where pixel(x,y) is the picture element of the image.

The inverse discrete cosine transform (IDCT) for a 2D signal of dimension NN is defined as follows:

$$pixel(x,y) = \frac{1}{\sqrt{2N}} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} C(i)C(j)DCT(i,j) \cos\left[\frac{(2x+1)i\pi}{2N}\right] \cos\left[\frac{(2y+1)j\pi}{2N}\right]$$

Where DCT(i,j) is the DCT coefficient and:

$$C(x) = \begin{cases} \frac{1}{\sqrt{2}} \cdots if \cdots x = 0 \\ 1 \cdots\cdots if \cdots x \rangle \cdot 0 \end{cases}$$

The transformation with DCT allows identifying those coefficients of signal information that can be removed without severe distortion of the image quality. But the drawback of the DCT as well as with all the transforms is the huge amount of computation even for small images because of the double sum in the formula. It is of the order of NxN. It is then time consuming and not practical for real image of 256x256 pixels, for example. To overcome this problem, the image is subdivided into small blocks (WINTZ 1972, JAIN 1989). Blocks of 64x64 will not result in a better compression than blocks of 16x16, although the computation time is higher.

### Properties of the DCT

Here is a summary of the properties of the DCT.

1. The DCT coefficients are real and orthogonal
   $C = C^*$ and $C^{-1} = C^T$ ($C^*$ is the complex conjugate of C).
2. The DCT is not the real part of the DFT.
3. The DCT has a fast version (FDCT). An N elements vector can be DCT transformed with O(NLog2N) operations via N-points FFT.
4. The DCT possesses an excellent energy compaction property for highly correlated data (JAIN 1989).
5. The DCT is very close to the KL transform for a stationary Markov sequence of first order.

## THE QUANTIZATION

The output matrix of the DCT uses even more memory space than the original image matrix. The pixels of the original image are coded with eight bits because the image has 256 gray levels but the output data fall within the interval of -1024 to +1024 and once coded, they require 11 bits.

The coefficients are quantized. An algorithm implements the procedure of quantization, which uses a quantization matrix where the large coefficients are coded with smaller quantization steps. The well-known formula is as follows (WINTZ 1972, ROSENFIELD 1982):

$$q(i,j) = \frac{DCT(i,j)}{Quantum(i,j)}$$

Where q(i,j) is the quantized value of the DCT.

q(i,j) is simplified to the nearest integer.

The quantization matrix is defined once the user introduces the desired factor of quality. Each element of the matrix is given by:

Quantum(i,j)=1+(i+j)Factor.of.quality

## CODING

The final stage of the compression process is the coding of the quantized values. This phase is performed in two steps:

- Rearrangement of the coefficients of the image according to a sequence in zigzag.
- Coding of the coefficients.

### Rearrangement of the Coefficients.

During the quantization phase, a great number of the coefficients of the DCT matrix are quantized to null values. This suggests to treat these null values differently from the other values of the coefficients. Instead of using Huffman or Arithmetic coding algorithms, we use the Run Length Encoding (RLE) algorithm, which gives the consecutive number of null values. A method to increase the length of such sequences is to rearrange the coefficients in zigzag manner. Instead of coding the coefficients naturally, row by row, the algorithm goes from one block to the other with a diagonal path as it is shown in Fig.3. In each block, the large values are treated first and then the small values.

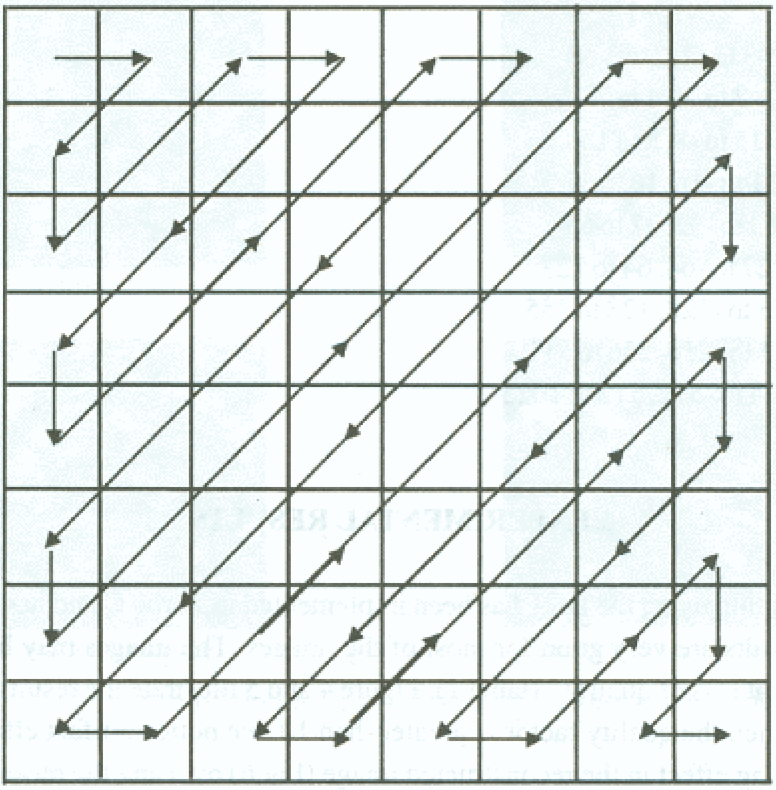


**Fig.3: The "zigzag" procedure for the block.**

## Entropy Coding

The algorithm converts the dc component of a block into a difference with the previous block dc value and then rearranges the block of the DCT to a zigzag sequence. The algorithm then treats the elements by using a mechanism of entropy coding. The coding sequence used is a combination of RLE and a coding of variable length integer. The output of the coder is a sequence of three repeated elements until the block is done. These three elements are:

- **The Sequence Length**: Number of consecutive zeros before the actual non-zero element in the DCT matrix.
- **The Bits Count**: Number of bits in the amplitude.
- **The Amplitude**: Amplitude of the coefficient of the DCT.

The values of the sequence length and of the bits count are combined to form an output code. The bits count is the number of bits used to code the amplitude. It is an integer. The bits count and the amplitude to be coded are given next:

| Bits count | Amplitude |
|---|---|
| 1 | -1, +1 |
| 2 | -3 to -2, 2 to 3 |
| 3 | -7 to -4, 4 to 7 |
| 4 | -15 to -8, 8 to 15 |
| 5 | -31 to -16, 16 to 31 |
| 6 | -63 to -32, 32 to 63 |
| 7 | -127 to -64, 64 to 127 |
| 8 | -255 to -128, 128 to 255 |
| 9 | -511 to -256, 256 to 511 |
| 10 | -1024 to -512, 512 to 1023 |

## EXPERIMENTAL RESULTS

The compression algorithm using the DCT has been implemented in Turbo C and has been tested on different sizes of images. The results are very good for most of the images. The images may be compressed at a high percentage without a great loss of quality (table 1). Figure 4 and 5 illustrate the results of a compression and a reconstruction cycle. When the quality factor is greater than 10, we notice artifact effects of the compression which appear as a blocking effect in the reconstructed image (Fig.6) by using the mean square error as defined by:

$$MSE = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m}\left[X(i,j)-\hat{X}(i,j)\right]^2$$

where X and are respectively the gray levels of the original image and the reconstructed image and mxm is the size of the image block.

The following table shows the compression gains as well as the MSE for image blocks of 8x8. Very high compression gains have been achieved with small degradation in the quality of the image.

| Quality Factor | Compression Gain % | bits per pixel | MSE |
|---|---|---|---|
| 1 | 63 | 3.02 | 3.8 |
| 2 | 74 | 2.12 | 6.2 |
| 3 | 80 | 1.68 | 7.9 |
| 4 | 83 | 1.42 | 9.2 |
| 5 | 85 | 1.26 | 10.2 |
| 10 | 89 | 0.89 | 13.4 |
| 15 | 91 | 0.75 | 15.9 |
| 25 | 92 | 0.64 | 17.4 |

**Table 1: Compression Results for the image << clown>> 320x200 with blocks of 8x8.**

Original Image

Quality <<2>> 2.212 bits/pixel

Quality <<10>> 0.89 bits/pixel

Quality <<25>> 0.64 bits/pixel

Fig. 4: Reconstruction of the image <<clown>> after one compression cycle

Original Image

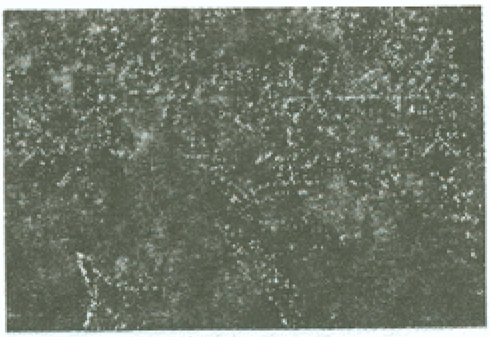
Quality <<7>> 0.87 bits/pixel

Quality <<15>> 0.70 bits/pixel

Quality <<25>> 0.62bits/pixel

Fig. 5: Reconstruction of the image <<portrait>> after one compression cycle

(a)



(b)

Fig.6: RMS Error on the image <<clown>> (a) quality <<3>> (b) quality <<25>>

## CONCLUSION

The DCT is an orthogonal transformation particularly suitable for image compression. It uses the important decorrelation property which, practically, results in the elimination of the unnecessary information (redundant information). An interesting aspect of the choice of the quantization matrix is to offer the user a great flexibility in the choice of image quality according to the compression rate. High compressions (low bit rates) have been achieved with relatively small degradation of the image quality.

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