COMPUTATIONAL MESHING TECHNIQUES FOR GENERATING CRACK-TIP ZONE STRESS ANALYSIS

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ABSTRACT. *This paper discusses the meshing techniques using finite element analysis to investigate the fracture phenomena of a standard compact tension model. The computed crack-tip stress intensity factor in comparison with experimental toughness, obtained via ASTM E399, is used as an indicator for evaluating the meshing accuracy. Free, controlled and localized meshing techniques are investigated using ANSYS 9.0 computational analysis has found that localized-meshing achieved the best accuracy in the stress intensity ratio at 0.96.*

KEYWORDS. Meshing, compact tension, crack-tip stress intensity, experimental toughness.

INTRODUCTION

Stress characterization of a geometry can be investigated using both analytical and numerical methods. Conventional analytical method appeals to physics and engineering principles to derive differential equations relating the variables of interest in a system. The principles include equilibrium, Newtonian laws, potential energy, strain energy, conservation of mass, Maxwell's equations, and so forth (Courant, 1943). However this method has limitation in solving excess number of variables. As a result, large safety factors have to be introduced in engineering designs that is not viable in both material usage and production activities.

In 1950s, researchers have found the solution to solve the mathematical difficulty in analytical method by transforming the differential equation approach into an algebraic problem, which is solved by computation software called the finite element method. This method partitions a structure into simply shaped portions called finite elements and generates an approximate solution for the variable of interest within each element (Lepi, 1998). The earliest finite element application has been applied in the aircraft industry (Barron, 1993). Since then the engineering field has been motivated to utilize the numerical method to provide a time-saving and less costly analysis.

One of the major features in finite element analysis is the mesh generation. It defines how the model will react to the loading conditions, material properties, and constraints (Araujo, *et al.*, 2000). ANSYS 9.0 code is equipped with 'automesher' that allows free-meshing in both two and three dimensional elements, the level options ranked according to the finery of mesh. As for fine meshing, particularly at the crack-tip zone, the controlled and localized-meshing are used, which concentrate element nodes at the area of interest. In term of element type, George (George, 1991) with his active researches claimed that the quadrilateral element is more preferable than the Delaunay triangular elements in 2-dimensional meshing.

MODEL CONFIGURATIONS

The adopted mathematical representation in this investigation is a compact tension model with fatigue pre-cracking loaded in uni-axial tension as shown in the Figure 1. The geometry a configuration is designed for plane strain conditions according to the standard test method coded E-399 (ASTM, 1990).

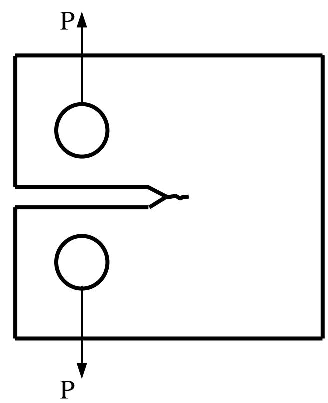


Figure. 1. Model configuration adopted for numerical analysis.

The material selected is made of low carbon steel, which consists of the following properties: elastic modulus 200 GPa, Poisson's ratio 0.30, and yield strength 358 MPa. The principle of LEFM concept is implemented in the computation, which yields the experimental fracture toughness in the form of Eqn. (1)

$$K_{IC} = \frac{P_Q}{BW^{1/2}}.\left(\frac{\left(2+a/W\right)\left(0.886+4.64a/W\text{-}13.32a^2/W^2+14.72a^3/W^3\text{-}5.6a^4/W^4\right)}{\left(1-a/W\right)^{3/2}}\right) \quad (1)$$

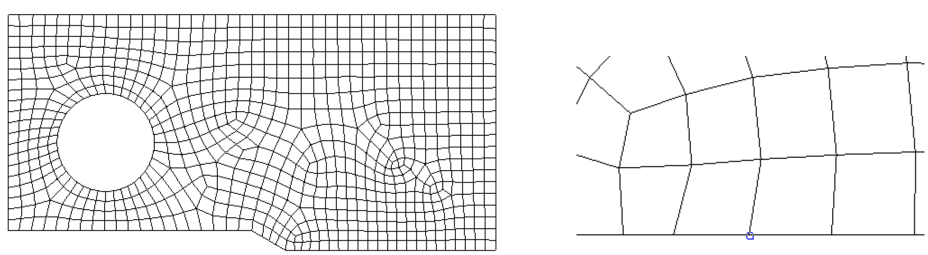
where P_Q is the acting force equal to 20kN, B, W and a are the thickness, width and crack length of model in meter (Lemaitre & Chaboche, 1990). The experimental toughness obtained is 32.586 MPa.m$^{1/2}$.

FINITE ELEMENT MESHING

The attention of this paper is confined to meshes of two-dimensional quadrilateral elements. Using ANSYS 9.0 code, a half model representing the symmetrical 2-dimensional plane is solved by three different meshing techniques. Each technique has its own unique characteristics in terms of number of elements used, Q_n, localized crack-tip behavior (represented by the radius for 1st row of elements, R_o), and the accuracy of crack-tip stress intensity ratio in percentage, $\%K_{num}/K_{exp}$.

Free-meshing

Free-meshing is automatically generated by the computational software upon the selection of fine-mesh level. There are few meshing levels available for selection, in which each level assigns specific Q_n, in accordance to the model geometry. For fine-meshing, where more elements are employed, longer computation time is required. The free-meshing has the characteristic of equally distributing the mesh-elements all over the numerical model. As a result, the mesh distribution at the crack-tip zone is similar to the areas remote from the tip. This homogeneous-type of meshing is shown in the Figure 2.

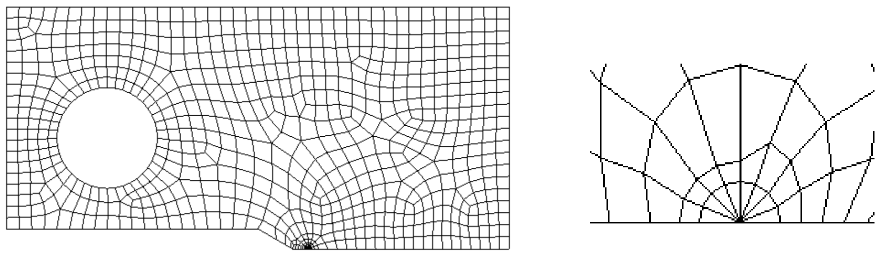


(a) **(b)**

Figure 2. Free-meshes of (a) half model and (b) region around the crack-tip.

Controlled-meshing

In controlled-meshing, it is desirable to have a concentration of relatively smaller elements in the region of greatest interest (Fenner, 1996). Unlike the free-meshing, the controlled-meshing is organized in such a way that the meshing initiates from the crack-tip. This technique constitutes the 'spider web' configuration, as claimed by Anderson (Anderson, 1995) as the most efficient mesh type for crack-tip stress analysis. It has the capacity to generate refined meshes about the vicinity of the crack-tip and construct meshes from fine to coarser degree as illustrated in the following Figure 3. The degree of meshes refinement lies upon the setting of R_o. In this case, the R_o analyzed are scaled from 5 to 500 microns.

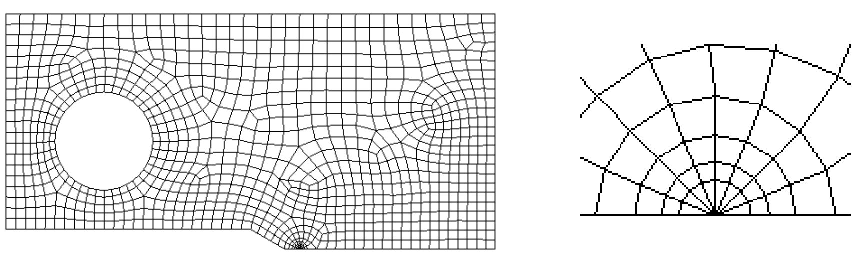


(a) (b)

Figure 3. Controlled-meshes of (a) half model and (b) region around the crack-tip.

Localized-meshing

The localized-meshing constitutes the similar meshing configuration characteristic approach as the controlled-meshing, with the additional of a localized-region created at the crack-tip. This localization characteristic enables the generation of supplementary organized fine meshes about the vicinity of the tip that structures better arrangement of nodes in comparison to the controlled-meshing. The R_o analyzed are also scaled from 5 to 500 microns. The meshing distribution is demonstrated in the Figure 4.



(a) (b)

Figure. 4. Localized-meshes of (a) half model and (b) region around the crack-tip.

RESULTS AND DISCUSSIONS

The three curves, as shown in the Figure 5, are the generated resultant characteristics for the meshing-types analyzed (free, controlled and localized meshing). The figure demonstrates the differences in the Q_n used for the computation of a half-model symmetrical analysis, with respect to the stress intensity ratio, K_{num}/K_{exp}. All three meshing-types produce very similar characteristic trend in the convergence of solutions. Generally, increasing trends of $\%K_{num}/K_{exp}$ are observed proportional to Q_n, which eventually converge to constant values, respectively shown in the figure.

The results for free, controlled, and localized meshing have converged at 712, 1162, and 939 of total elements, respectively. Free-meshing has employed the least elements in finite element meshing compared to the controlled and localized techniques, which generates a poor result in the convergences of K_{num}/K_{exp} at about 43%. In contrast, the controlled-meshing has employed the most elements with convergence value of K_{num}/K_{exp} achieving at 68%. While in the localized meshing technique the K_{num}/K_{exp} accuracy at 96% is obtained. Further more, localized-meshing solution is generated at a lesser Q_n compared to controlled-meshing. The advantage of meshing with higher Q_n does not necessarily produce a better result for generating the crack-tip zone stress analysis. Therefore, the localized-meshing has proven that very refine localized analysis at the crack-tip boundary is necessary to attain higher accuracy of K_{num}/K_{exp}.

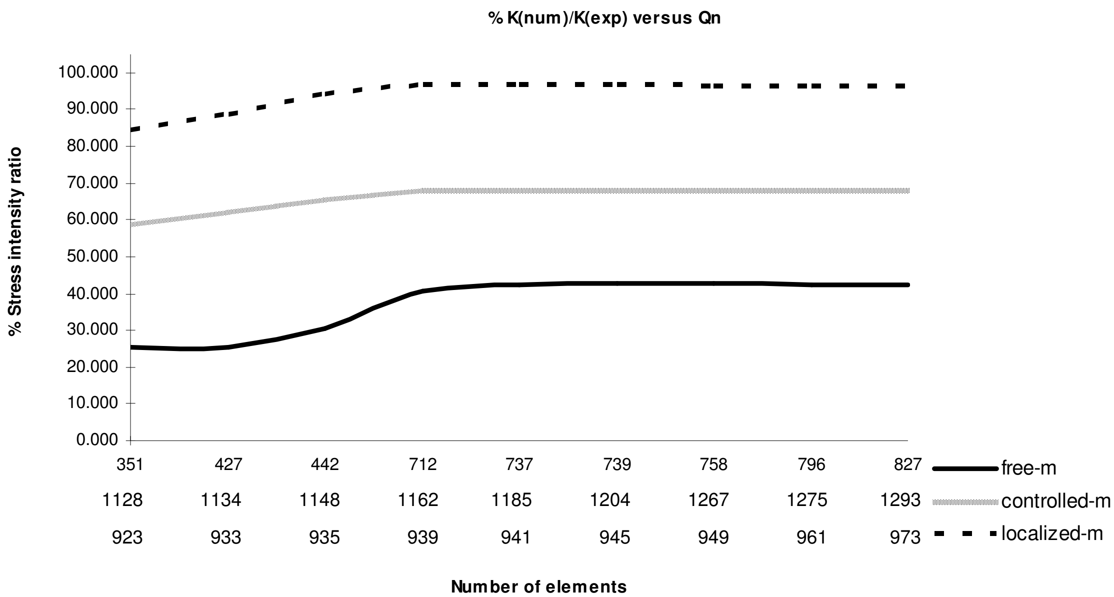


Figure 5. Characteristic curves for free-, controlled- and localized-meshing in accordance to Q_n.

Further characterization of the crack-tip behavior through the variation of R_o can also be observed. How small the elements at the vicinity of crack-tip can be defined, with acceptable accuracy, which determines the degree of local crack-tip zone to be analyzed. In this case, only the controlled-meshing and localized-meshing exhibit this characterization option. The free-meshing is generated automatically by the program code without crack-tip mesh concentration, and at large element size with almost equal distribution throughout the global geometry. The remaining two meshing-types are capable of performing crack-tip zone meshing up to R_o of 5 microns about the crack-tip. Therefore, a resolved characteristic is produced in the Figure 6, demonstrating that the localized-meshing has produced a far more promising result than the controlled-meshing.

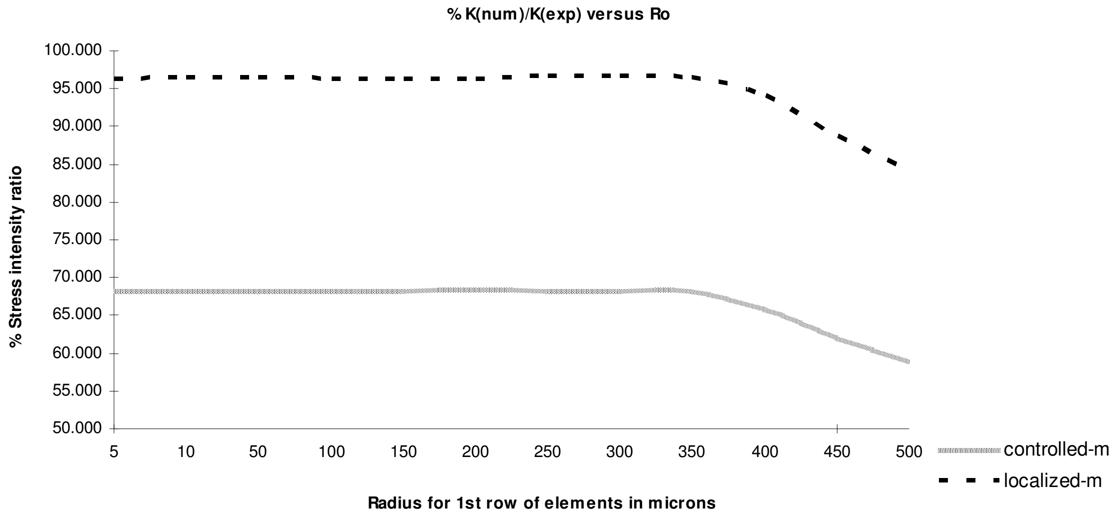


Figure 6. Characteristic curves for controlled-meshing and localized-meshing in accordance to R_o.

CONCLUSION

The localized-meshing technique has shown to be a better approach for designing and analyzing both larger and micro scale fracture phenomena at the vicinity of the crack-tip zone. Therefore, it has also provided the advantage over the large scale or global mesh analysis for the important investigation and characterization of local crack-tip plastic zone behavior.

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