A NEW NUMERICAL ALGORITHM FOR ANTENNA ANALYSIS

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ABSTRACT. *The Finite-Difference Time-Domain (FDTD) method is very popular in analyzing an antenna due to its simplicity and robustness of implementation. In this paper, we adapt a new concepted to the FDTD method. To show the efficiency of the new method, we compared the new method to the original FDTD method to solve a one-dimensional problem. From the result, it shows that the new method solves the problem faster (67%) and give approximately the same result compared to the standard FDTD method.*

KEYWORDS. *Finite-Difference Time-Domain (FDTD), high speed low order FDTD, antenna analysis.*

INTRODUCTION

Radio waves contribute to high quality of modern life and comfort. Electric fields propagate in the form of radio wave generated from an antenna of a communication device is the main event in wireless communication. Advances in communication industry, need the devices to be design carefully. Smaller built-in antenna with broader bandwidth has to be designed to fulfill these needs, e.g. the Planar Inverted F antenna (PIFA). To assist a low-cost design of an antenna, there is a need for “soft” tools that can simulate the behavior of the electric fields or radiation generated from the antenna. The behavior of the fields can be predicted by using partial differential equations.

The finite-difference time-domain (FDTD) method was introduced by Yee (1966) for solving Maxwell equations in time-domain. The method further was developed by Taflove (2000). The method represents a simple and efficient approach of solving Maxwell equations in differential time-domain form. The formula proposed by Yee needs the electric and magnetic fields to be solved alternately from starting point to the required time step.

The method is explicit in nature and depends on Courant-Frederich-Levy (CFL) condition to be stable. Even though the standard FDTD method is a very credible and precise method, there are still drawbacks in the method. FDTD needs large amount of memory and long processing time (Araujo, *et al.,* 2003).

An ordinary approach to improve the speed of the algorithm would be using more powerful computers, computers with several processors working concurrently, cluster of workstations or by pile's of personal computers, which known as Beowulf cluster (Rodohan

& Saunders, 1994; Jensen, *et al.,* 1994; Nguyen *et al.*, 1994; Fijany, *et al.*, 1995; Schiavone, *et al.*, 2000; Guiffaut & Mahdjoubi, 2001; Zhenghui, *et al.*, 2002; Yang, *et al.*, 2003; Araujo, *et al.*, 2003; Yu, *et al.*, 2004). Another popular approach is by solving the same problem with coarser grid but using higher-order method (Georgakapoulus, *et al.*, 2002; Lan, *et al.*, 1999; Taflove, 2000).

In this paper, we proposed a new concept to be applied in designing FDTD algorithm. This new method, which is called high-speed low-order(m) finite-difference time-domain (HSLO(m)-FDTD), is implemented in a slightly different algorithm from the standard FDTD.

THE NEW METHOD

The development of this method is inspired by Modified Explicit Group (MEG) introduced recently by Othman and Abdullah (2000) to solve Poisson problem. The MEG method is actually an extension of the concept of the half-sweep iterative method, which is proposed by Abdullah (1991) through the Explicit Decoupled Group (EDG) iterative method in solving the same Poisson problem.

In this paper we apply the concept used in MEG to the FDTD but without iterative process to solve Maxwell equations. The iterative process has to be ignored because of the explicit nature of the discrete formulations.

In 1864, James Clerk Maxwell introduced a couple of equations, called Maxwell equations that are still being applied till today. This equations has been used by researchers from all over the world to design aircraft, antenna, circuit-board, cellular phones, bio-electromagnetic system, digital circuit, scattering problem and wave propagation (Kunz & Luebber, 1993; Taflove, 1995; Taflove & Brodwin, 1975;Yee, 1966). The Maxwell equations in free space is given by

Ampere's law: $$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0}\nabla \times H, \tag{1}$$

Faraday's law: $$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0}\nabla \times E. \tag{2}$$

where ε_0, μ_0, E and H are the electric permittivity, magnetic permeability, electric and magnetic fields, respectively. For the one-dimensional case using electric in x-direction, E_x and magnetic in y-direction, H_y, Eqs. (1) and (2) become

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0}\frac{\partial H_y}{\partial z}, \tag{3}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z}. \tag{4}$$

These are the equations of a plane wave with the electric field oriented in the x-direction, magnetic field in the y- direction, and traveling in the z-direction. The right hand side of Eqs. (3) and (4) above is then approximated by Taylor series at $\frac{m}{2}\Delta z$ as below

$$f\left(z_i+\frac{m}{2}\Delta z\right)=f(z_i)+\frac{m}{2}\Delta z f^1+\left(\frac{m}{2}\Delta z\right)^2\frac{f^2}{2!}+\cdots \tag{5}$$

$$f\left(z_i-\frac{m}{2}\Delta z\right)=f(z_i)-\frac{m}{2}\Delta z f^1+\left(\frac{m}{2}\Delta z\right)^2\frac{f^2}{2!}+\cdots \tag{6}$$

where m is any odd number ranging from 1 to 7 and for the left hand side of Eqs. (3) and (4) will be approximated by Taylor series at $\frac{1}{2}\Delta t$ as below

$$f\left(t_i+\frac{1}{2}\Delta t\right)=f(t_i)+\frac{1}{2}\Delta t f^1+\left(\frac{1}{2}\Delta t\right)^2\frac{f^2}{2!}+\cdots \tag{7}$$

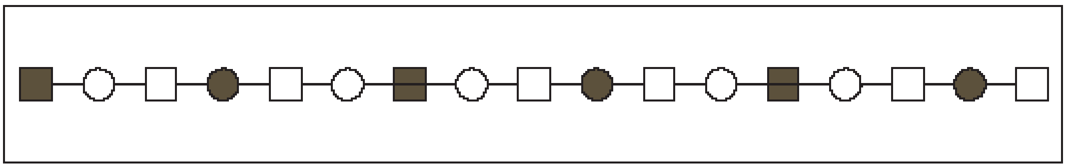
$$f\left(t_i-\frac{1}{2}\Delta t\right)=f(t_i)-\frac{1}{2}\Delta t f^1+\left(\frac{1}{2}\Delta t\right)^2\frac{f^2}{2!}+\cdots \tag{8}$$

By using Eqs. (5), (6), (7) and (8) to approximate (3) and (4), yields

$$\tilde{E}_x^{n+\frac{1}{2}}(k)=\tilde{E}_x^{n-\frac{1}{2}}(k)-\frac{\Delta t}{m\Delta x\sqrt{\varepsilon_o\mu_o}}\left(H_y^n\left(k+\frac{m}{2}\right)-H_y^n\left(k-\frac{m}{2}\right)\right), \tag{9}$$

$$H_y^{n+1}\left(k+\frac{m}{2}\right)=H_y^n\left(k+\frac{m}{2}\right)-\frac{\Delta t}{m\Delta x\sqrt{\varepsilon_o\mu_o}}\left(\tilde{E}_y^{n+\frac{1}{2}}(k+m)-\tilde{E}_y^{n+\frac{1}{2}}(k)\right). \tag{10}$$

The alternating calculation of Eqs. (9) and (10) are the main ingredient in HSLO(m)-FDTD method. In this paper we will only discuss when m=3 (since we used *3h* spatial discretization) and will be acronym with HSLO(3)-FDTD. Formulation for FDTD can be gathered when m=1. For detail on FDTD formulation, please refer to Yee (1966). Eqs. (9) and (10) will be executed only at the black nodes in the computational displayed in Figure 1.



Fiqure 1. Computational domain for HSLO(3)-FDTD.

The remaining white nodes will be executed outside the main loop by using linear weighted interpolation.

NUMERICAL EXPERIMENTS AND RESULTS

In order to evaluate the performance of HSLO(3)-FDTD, we numerically solve the one-dimensional Maxwell's equation with 400MHz of frequency with Gaussian Pulse transmitted 1 meter from a monopole antenna in two opposite direction. We used a perfectly electric conducting at both boundaries. We discretize the computational domain into 200 grid points, i.e. 2.67λ and the pulse is generated at the middle of the computational domain. The experiment was run on SMP machine (Sun Fire V1280) but using only one of its processors.

To show the gain of speed obtained by HSLO(3)-FDTD method, electric fields simulated by HSLO(3)-FDTD method and an existing standard FDTD method and their execution time are made and compared. The results of electric fields simulation are as in Figures 2 and 3, which show the behavior of wave propagation from the point source to both directions. The calculation time for HSLO(3)-FDTD and standard FDTD are shown in Figure 4. The result shows that HSLO(3)-FDTD has reduced the calculation time of FDTD by 50%-60%. The reduction will further reduce (until 67%) at higher time steps (time step approaching ∞). The maximum reduction of calculation time can be estimated by maximum relative gain in complexity of arithmetic as in Table 1.

Table 1. Comparison of complexity of FDTD and HSLO(3)-FDTD (N_p number of grid points, N_t number of time steps)

Method	ADD/SUB	MUL/DIV
FDTD	$4N_pN_t$	$2N_pN_t$
HSLO(3)-FDTD	$(4/3)N_pN_t + (2/3)N_p$	$(2/3)N_pN_t + (2/3)N_p$

From Table 1, we can calculate relative gain by HSLO(3)-FDTD to standard FDTD. Formulation of the relative gain for both addition/subtraction (ADD/SUB) and multiplication/division (MUL/DIV) as below.

$$\text{Relative gain (ADD/SUB)} = \lim_{N_t \to \infty} \left(\frac{4N_pN_t - \left(\frac{4}{3}N_pN_t + \frac{2}{3}N_p\right)}{4N_pN_t} \right) = \lim_{N_t \to \infty} \left(\frac{2}{3} - \frac{1}{6N_t} \right) = \frac{2}{3}.$$

Taking the limit $N_t \rightarrow \infty$, we obtain the percentage gain of 67% and

$$\text{Relative gain (MUL/DIV)} = \lim_{N_t \to \infty} \left(\frac{2N_pN_t - \left(\frac{2}{3}N_pN_t + \frac{2}{3}N_p\right)}{2N_pN_t} \right) = \lim_{N_t \to \infty} \left(\frac{2}{3} - \frac{1}{3N_t} \right) = \frac{2}{3}.$$

Again by taking the limit $N_t \rightarrow \infty$, we also obtain the percentage gain of 67%. As the complexity is the major contributor to algorithm processing time, we predict that the maximum relative reduction in calculation time for HSLO(3)-FDTD to standard FDTD is 67%.

The accuracy of the HSLO(3)-FDTD are check by global and the amplitude error and are summarize in Table 2. The percentage of wave phase velocity error for FDTD and HSLO(3)-FDTD are 0.4% and 3.6% respectively. This result is still acceptable and at par with other research output (Lan *et al*., 1999).

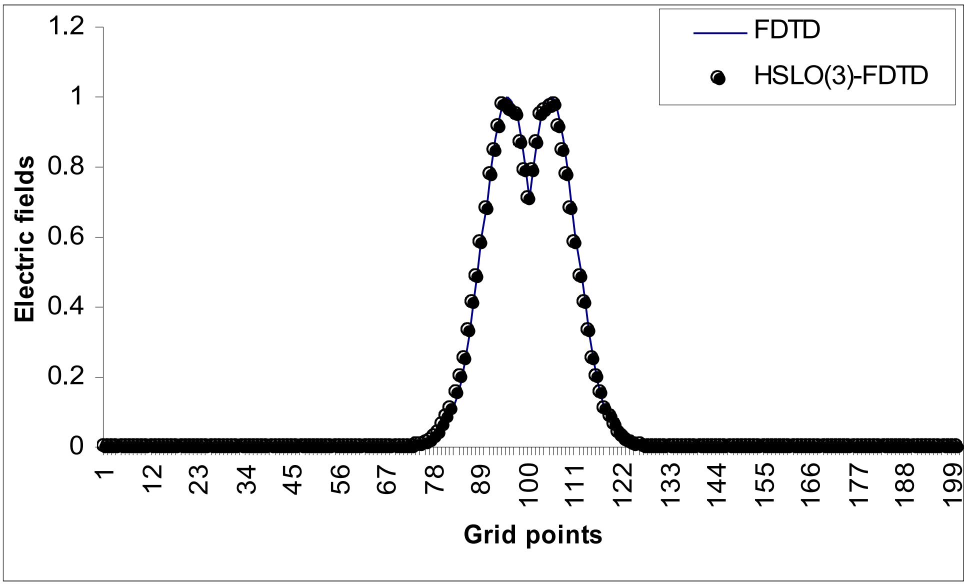


Figure 2. Electric fields in volt/meter at 1.1 ns generated by a point source in the middle of computational domain.

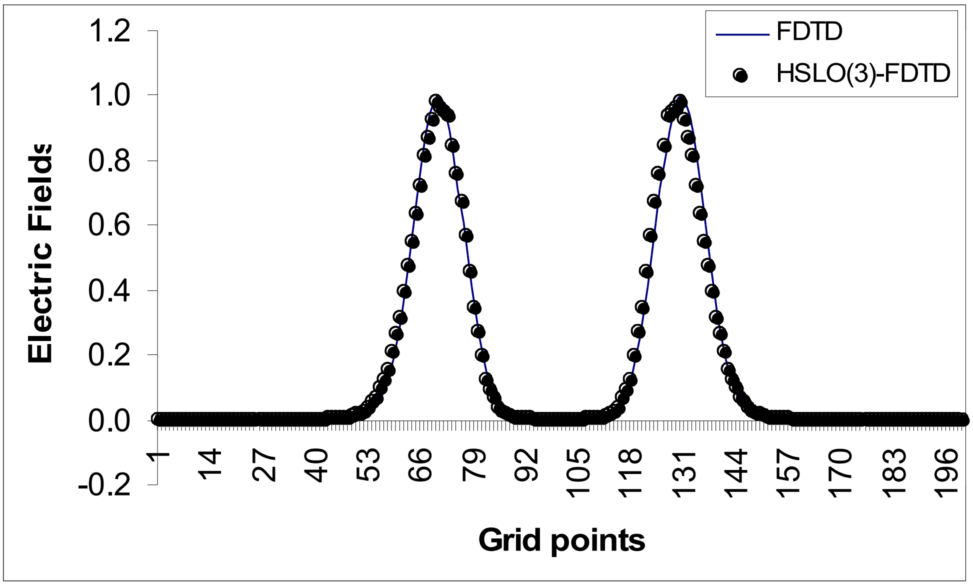


Figure 3. Electric fields in volt/meter at 2.2 ns generated by a point source in the middle of computational domain.

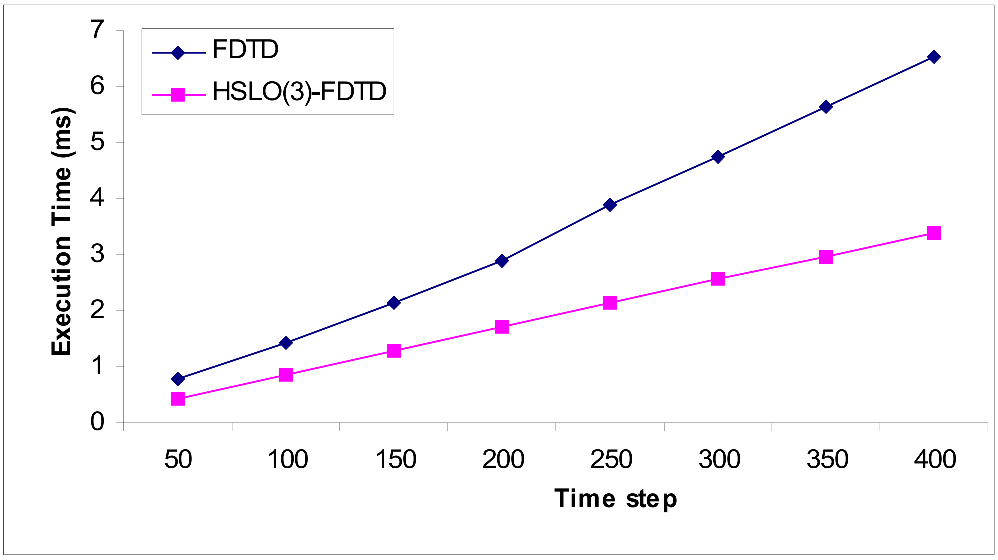


Figure 4. Comparison of calculation time between HSLO(3)-FDTD and standard FDTD.

Table 2. Global and amplitude errors of HSLO(3)-FDTD compared to FDTD.

Error	Time Step 50	Time Step 100	Time Step 150
Global Error	2.35e-7	1.40e-6	3.77e-6
Amplitude Error	2.076e-2	2.321e-2	3.130e-2

CONCLUSION

This paper presents a new solver in time-domain solution of Maxwell equations. The performance of this scheme was tested for a problem in one dimension for antenna analysis. The major advantages of this method are that it has less complexity and solve faster than the existing FDTD scheme. The results above confirm the advantages that can be derived from the HSLO(3)-FDTD method. It is clearly shown that the new method offers as a merit-able alternative direct scheme to solve real time problems. Our future research will concern on analyzing the behavior of HSLO(m)-FDTD method when $m=5$ and 7. We will also extend this method to solve two dimensional problem.

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