

## ORIENTATION MODEL FOR STRAIGHT ARC WELDING-PATH PLANNING

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**ABSTRACT.** *This paper presents a mathematical method to constrain the frame of welding torch based on the inputs of transverse angle and longitudinal angle for performing the straight arc. Locations of 4 points are required to define the job surfaces and welding direction. The torch frame is then modelled and can be solve analytically or numerically for the orientation matrix of the torch with respect to the reference frame. Hence, upon the determination of the torch orientation, straight arc welding trajectory can be automatically planned.*

**KEYWORDS.** Orientation Matrix, Robotic Arc Welding, Torch Orientation, Welding Torch Frame Assignment.

### INTRODUCTION

A quality weld in robotic arc welding is made of proper synchronisation and consistency any manipulatable welding inputs over the unmanipulatable inputs, which are governed by three major equipments namely robot manipulator, electrode wire feeder and welding power source. The manipulatable inputs include the torch speed, torch angle, wire feed rate, voltage, and etc. Whereas for the unmanipulatable inputs include the joint geometry, and the plate thickness of a work piece (Dornfeld, *et al.*, 1982).

Nonetheless, online manual programming is still favourable in the industries to mimic the welding motion due to the difficulties in modelling the process for handling all the inputs. Hence, there is no real system that is capable of automatically plan the welding process.

Besides the welding torch position, orientation of the welding torch is one of the many important features needed to be controlled, which consists of two angles normal to each other; (i) transverse angle and (ii) longitudinal angle (Sicard and Levine, 1988). These angles can affect the penetration, size and shape of the weld bead (Cary, 1995). As a whole, the welding process can be defined as a five degree-of-freedom (5-DOF) process regardless of the rotational DOF about its electrode axis, which is not relevant or has been constrained.

This paper, therefore, concentrates in obtaining the torch orientation matrix using vectors based on the transverse angle and welding direction from the joint geometry. The welding torch orientation with user-define longitudinal angle is used as a part of the welding process model in torch trajectory planning.

The torch frame is introduced and constrained in such a way that the torch handle will always be on the plane that split the two job surfaces equally into a straight welding joint. Four point-locations, which can be retrieved via online-sensors, provide the essential joint geometry and welding direction feedbacks for immediate action or correction.

### TORCH FRAME ASSIGNMENT

As any other end effector of robot manipulator concerns, the last coordinate system should be assigned on the welding torch at a functional point. For this reason, the origin of the torch frame  $O_n$  is assigned at the tip of the torch since it is the point where the arc welding is taking place.

Figure 1 illustrates on how the  $n$ -th frame is assigned at the torch. As usual, the normal axis ( $n$ -axis), sliding axis ( $s$ -axis) and approach axis ( $a$ -axis) are introduced to explicitly label the perpendicular  $x_n$ ,  $y_n$  and  $z_n$  axes, respectively,  $n$ -suffix being the  $n$ -th frame.

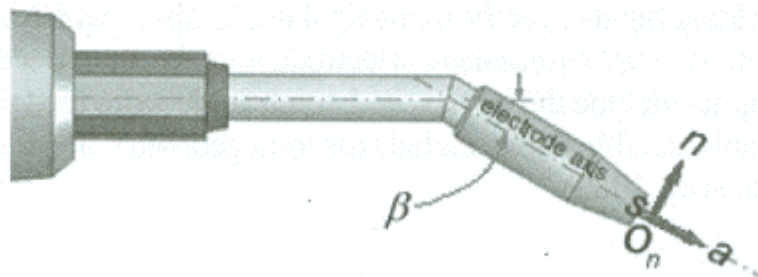


Figure 1. Torch frame assignment.

The  $a$ -axis is aligned to the electrode axis and points out from the torch nozzle. The  $s$ -axis, which represents the mobility on sideways direction of the torch, is set such that it is perpendicular and pointing out from this paper. The  $n$ -axis is then set intuitively according to the right-hand-rule. These axes are perpendiculars of each other.

Since the torch frame- $(n-1)$  relative to the last joint frame- $n$  is fixed, the transformation matrix of the torch frame with respect to the last joint frame can then be determined prior to the torch trajectory planning.



## TORCH ORIENTATION ON WELDMENT

In the 3-dimensional world, the torch tip frame is defined via three position inputs and three orientation inputs. The position inputs ( $x_n, y_n, z_n$ ) can be easily supplied by the linear interpolation of straight welding joint-path with respect to the constant torch speed on the time basis. Thus, the orientation inputs of the  $n$ -th frame are only governed by the transverse and longitudinal angles,  $(\varphi, \lambda)$  respectively. While a rotational DOF of the torch tip frame is set to redundant or constrained for a simple case.

In order to simplify the problem, the redundant DOF is constrained during the arc weld process. Assigning this constrain means that there is no rotation about the electrode axis along the welding path. This simplified case is particularly true for straight arc welding.

Hence, the torch has to be set such that the torch-handle is always at the mid-plane, determined by the electrode axis and the weld axis, which split the angle of weld  $2\varphi$  equally about a straight welding joint. While the angle of weld refers to the angle that encompasses the weld between the two job surfaces measured on a cutting-plane normal to the joint path. Thus, the  $s$ -axis is perpendicular to the mid-plane along the process path as shown in Figure 2.

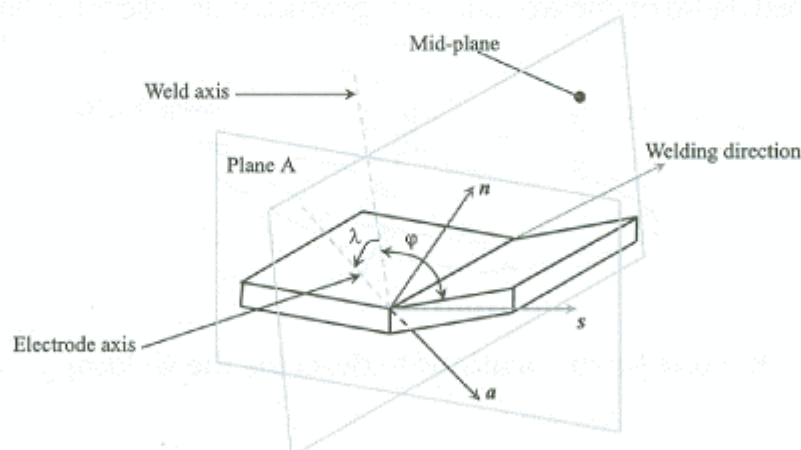


Figure 2. Torch frame on welding job.

At the same time, an  $a$ -axis is set to point out from the torch nozzle along the electrode axis that makes an angle from the weld axis on the mid-plane. This angle is referred as the longitudinal angle  $\lambda$  (also known as travel angle), which is user-defined and has the effects on the penetration and size of the weld bead (Cary, 1995). The angle  $\lambda$  is always less than  $90^\circ$ . To avoid ambiguity, the longitudinal angle is denoted as the push angle with a positive sign when the torch is pointing in the direction of the welding. In contrast, the longitudinal angle is denoted as the drag angle when a negative sign assigned.

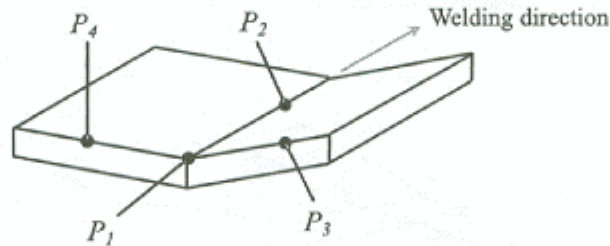
## MATHEMATICAL MODEL

In order to adapt the torch precisely to the welding inputs, its orientation must be able to be formulated into a useful form. Orientation matrix is an important form found in most robotic literatures, which contains the relationship between two respective frames in a  $3 \times 3$  matrix. In this case, the orientation matrix is expressed by the components breakdown of torch tip frame,  $tp$  with respect to the reference frame,  $r$  as denoted by  ${}^rR_{tp}$ , as shown in Equation (1) below.

$${}^rR_{tp} = \begin{bmatrix} i_x & j_x & k_x \\ i_y & j_y & k_y \\ i_z & j_z & k_z \end{bmatrix} \quad (1)$$

The notations  $i$ ,  $j$  and  $k$  denotes the unit vectors in moving  $n$ -,  $s$ - and  $a$ -axis of the torch tip frame, while the notations  $x$ ,  $y$  and  $z$  are denoting the fixed  $x$ -,  $y$ - and  $z$ -axis of the reference frame in a spatial space.

Based on the definition, the unit vectors of the standard orthogonal  $n$ -,  $s$ - and  $a$ -axis of the torch tip frame are required to be defined accordingly to the welding joint and welding angles in which is again described based on the welding joint geometry, as described earlier in Section 3.



**Figure 3. Four point-locations to describe the welding joint.**

Therefore, a four point-locations coordinates are necessary to describe the welding joint geometry (see Figure 3). A vector from point  $P_1$  to  $P_2$  along the welding joint direction is then denoted such that:

$$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle = \langle x_{12}, y_{12}, z_{12} \rangle \quad (2)$$

where

$x_i$  =  $x$ -coordinate of point  $P_i$

$y_i$  =  $y$ -coordinate of point  $P_i$

$z_i$  =  $z$ -coordinate of point  $P_i$



Similarly, two vectors describing point  $P_3$  with respect to  $P_1$  and point  $P_4$  with respect to  $P_1$  is denoted such in Equation (3) and Equation (4):

$$\overrightarrow{P_1P_3} = \langle x_{13}, y_{13}, z_{13} \rangle \quad (3)$$

$$\overrightarrow{P_1P_4} = \langle x_{14}, y_{14}, z_{14} \rangle \quad (4)$$

From these vectors, the two surfaces of the job are then mathematically described by determining the normal to the mentioned planes. Using the cross product rule, the normal to the plane containing points  $P_1, P_2$  and  $P_3$  can be obtained as in Equation (5):

$$n_1 = \overrightarrow{P_1P_3} \times \overrightarrow{P_1P_2} \quad (5)$$

$$= \begin{vmatrix} i & j & k \\ x_{13} & y_{13} & z_{13} \\ x_{12} & y_{12} & z_{12} \end{vmatrix}$$

$$= Ai + Bj + Ck$$

where

$$\begin{aligned} A &= y_{13} \cdot z_{12} - y_{12} \cdot z_{13} \\ B &= x_{12} \cdot z_{13} - x_{13} \cdot z_{12} \\ C &= x_{13} \cdot y_{12} - x_{12} \cdot y_{13} \end{aligned}$$

Similarly, the normal to the plane that contains the points  $P_1, P_2$  and  $P_4$  can be obtained as in Equation (6):

$$\begin{aligned} n_2 &= \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_4} \\ &= Di + Ej + Fk \end{aligned} \quad (6)$$

where

$$\begin{aligned} D &= y_{12} \cdot z_{14} - y_{14} \cdot z_{12} \\ E &= x_{14} \cdot z_{12} - x_{12} \cdot z_{14} \\ F &= x_{12} \cdot y_{14} - x_{14} \cdot y_{12} \end{aligned}$$

From these normal equations, the angle between them,  $\tilde{\alpha}$ , which represents the inclination angle of one job surface from the other, is computed using the dot product:

$$\gamma = \cos^{-1} \left( \frac{n_1 \cdot n_2}{|n_1| |n_2|} \right) \quad (7)$$

Therefore, the transverse angle  $\Phi$  that split the angle of weld equally into two can be obtained as:

$$\varphi = \frac{\pi - \gamma}{2} \quad (8)$$

Next, a plane (Plane A) and a point ( $P_5$ ) are to be introduced. The Plane A has the normal of the vector that lies along the welding joint  $\overrightarrow{P_1P_2}$ . Whereas, the point  $P_5$  is set to lie along the weld axis, which made up of the intersection of Plane A and mid-plane of the two surfaces. From the geometrical relation, the vector  $\overrightarrow{P_1P_5}$  is observed to be always perpendicular to the  $\overrightarrow{P_1P_2}$  and therefore, yielding to Equation (9):

$$\begin{aligned} \overrightarrow{P_1P_2} \cdot \overrightarrow{P_1P_5} &= 0 \\ x_{12} \cdot b_x + y_{12} \cdot b_y + z_{12} \cdot b_z &= 0 \end{aligned} \quad (9)$$

Also, since  $\overrightarrow{P_1P_5}$  is lying on the mid-plane, it makes an angle of  $\varphi$  from  $\overrightarrow{P_1P_3}$  such that:

$$\begin{aligned} \overrightarrow{P_1P_3} \cdot \overrightarrow{P_1P_5} &= |\overrightarrow{P_1P_3}| |\overrightarrow{P_1P_5}| \cos \varphi \\ \overrightarrow{P_1P_3} \cdot \overrightarrow{P_1P_5} &= |\overrightarrow{P_1P_3}| \cos \varphi \\ x_{13} \cdot b_x + y_{13} \cdot b_y + z_{13} \cdot b_z &= \sqrt{x_{13}^2 + y_{13}^2 + z_{13}^2} \cos \varphi \end{aligned} \quad (10)$$

However, these two equations will only yield to the weld axis equation and are not sufficient to solve for the  $b_x$ ,  $b_y$  and  $b_z$ . So,  $\overrightarrow{P_1P_5}$  is later initialized as a unit vector; since we are only dealing with unit vectors for modelling the orientation of the orthogonal torch tip frame:

$$\begin{aligned} |\overrightarrow{P_1P_5}| &= \sqrt{b_x^2 + b_y^2 + b_z^2} = 1 \\ b_x^2 + b_y^2 + b_z^2 &= 1 \end{aligned} \quad (11)$$

Equations (9), (10) and (11) consist of a set of non-linear equation, which can be solved numerically using Newton's method for nonlinear systems (Faires and Burden, 2003). From equation (11), one should notice that  $b_x$ ,  $b_y$  and  $b_z$  are in the bound of  $[-1, 1]$ . Thus, initial approximation for the suggested numerical method can be set to 1.

Now, to construct the  $s$ -axis of the tip of the torch, a unit vector  $\overrightarrow{P_1P_6}$  is initiated as its representation. Therefore, it has the magnitude of unity:

$$|\overrightarrow{P_1P_6}| = 1 \quad (12)$$



According to the property described for  $s$ -axis, the  $\overrightarrow{P_1P_6}$  is normal to the mid-plane of the two surfaces of the work piece. Thus, it is perpendicular to  $\overrightarrow{P_1P_2}$  and  $\overrightarrow{P_1P_5}$  such that:

$$\overrightarrow{P_1P_2} \cdot \overrightarrow{P_1P_6} = 0 \quad (13)$$

$$\overrightarrow{P_1P_5} \cdot \overrightarrow{P_1P_6} = 0 \quad (14)$$

Upon solving the Equations (12), (13) and (14) simultaneously will yield to the component of the unit vector  $\overrightarrow{P_1P_6}$  that is denoted as  $\langle j_x, j_y, j_z \rangle$ .

In the similar way, a unit vector  $\overrightarrow{P_1P_7}$  is used to represent the  $a$ -axis of the tip of the torch. Hence,

$$|\overrightarrow{P_1P_7}| = 1 \quad (15)$$

As an orthogonal frame,  $\overrightarrow{P_1P_7}$  must be normal to  $s$ -axis given by the unit vector yielding to:

$$\overrightarrow{P_1P_6} \cdot \overrightarrow{P_1P_7} = 0 \quad (16)$$

Subsequently, by incorporating the desired longitudinal angle of the torch  $\lambda$ , the vector makes  $\overrightarrow{P_1P_7}$  an angle of  $(\lambda + \pi)$  from  $\overrightarrow{P_1P_5}$  on the mid-plane, which yields the following relation:

$$\begin{aligned} \overrightarrow{P_1P_5} \cdot \overrightarrow{P_1P_7} &= |\overrightarrow{P_1P_5}| |\overrightarrow{P_1P_7}| \cos(\lambda + \pi) \\ \overrightarrow{P_1P_5} \cdot \overrightarrow{P_1P_7} &= -\cos \lambda \end{aligned} \quad (17)$$

Therefore, solving Equations (15), (16) and (17) simultaneously result to the components of the  $a$ -axis is denoted by  $\langle k_x, k_y, k_z \rangle$ .

The  $n$ -axis of the tip of the torch, which is represented by another unit vector  $\overrightarrow{P_1P_8}$  with its components of  $\langle i_x, i_y, i_z \rangle$ , can be derived intuitively based on the right-hand rule, provided the  $y$ - and  $z$ -axis are known, such that the following condition applies:

$$\overrightarrow{P_1P_6} \times \overrightarrow{P_1P_7} = \overrightarrow{P_1P_8}, \quad (18)$$

The Equation (18) is expanded as follows:

$$\begin{vmatrix} i & j & k \\ j_x & j_y & j_z \\ k_x & k_y & k_z \end{vmatrix} = \begin{vmatrix} i_x & i_y & i_z \end{vmatrix} \quad (19)$$

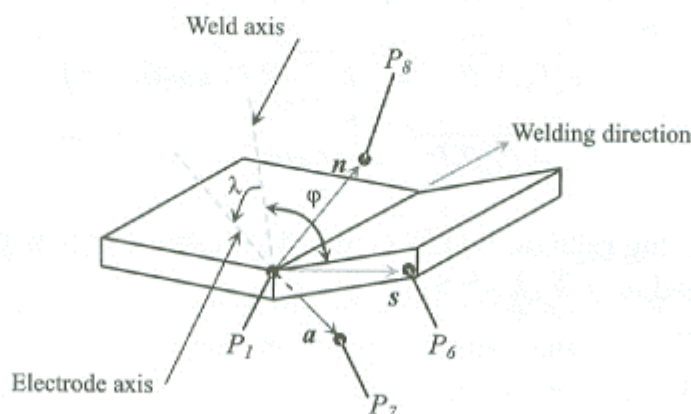
Therefore, we obtain the following

$$i_x = \begin{vmatrix} j_y & j_z \\ k_y & k_z \end{vmatrix} \quad (20)$$

$$i_j = \begin{vmatrix} j_z & j_x \\ k_z & k_x \end{vmatrix} \quad (21)$$

$$i_z = \begin{vmatrix} j_x & j_y \\ k_x & k_y \end{vmatrix} \quad (22)$$

Thus, solving Equation (20), (21) and (22) simultaneously yields to  $\langle i_x, i_y, i_z \rangle$  that completes the orthogonal frame of the tip of the torch governed by the transverse and longitudinal angle of the torch during welding operation.



**Figure 4. Three unit vectors to represent the torch tip frame.**

Having the components of the  $n$ -,  $s$ - and  $a$ -axis at the tip of the torch being evaluated, the orientation matrix of the frame with respect to the reference frame in Eqn. (1) can now be formed.



## DISCUSSION

Since the coordinate of the points  $P_3$  and  $P_4$  are taken without any guide or constraint as long as they appear to represent the appropriate two job surfaces, an intermediate vector  $\overline{P_1P_3}$  has been introduced to free equations (12) - (22) from total dependency on vector  $\overline{P_1P_3}$  and  $\overline{P_1P_4}$  in the derivation. However, the assignment of point  $P_3$  and  $P_4$  must be strictly followed as illustrated in Figure 3 such that point  $P_3$  is on the right side of the welding joint and point  $P_4$  is on the left side of the welding joint. This is due to the assignment of these points being important to define the opening of the angle of weld.

Mathematically, the model has been successfully derived in Section 4. It has completely developed the required orientation matrix by solving a non-linear system simultaneously via numerical method.

## CONCLUSION

Control over welding parameters such as longitudinal and transverse angle depends on torch orientation. Coordinates of 4 points, which can be retrieved by sensors, to describe torch orientation, welding direction and two job surfaces are applied to form adequate vectors. By geometric relation of these vectors to the unit vectors of the orthogonal torch tip frame, equations are formed and solved simultaneously for the components of the  $n$ -,  $s$ - and  $a$ -axis with respect to  $x$ -,  $y$ - and  $z$ -axis. The solutions will then form the elements in the orientation matrix for practical application. Therefore, this mathematical model allows computational solution of orientation matrix for automatic path or trajectory planning in robotic arc welding.

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