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DESIGN OF AXIALLY-DISCONTINUOUS THIN-WALLED FRAME STRUCTURES FOR COMBINED BENDING AND TORSION: Application Of One Dimensional Beam Elements Via A Modified Beam Element And A Joint Element

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ABSTRACT. *A thin-walled open section warps when subjected to torsion. If connected to another section at an angled joint, this warping causes not only twisting of the second section but also bending. The latter mode of deformation primarily occurs about that of the weaker axis.*

In practice, warping is usually restrained at sufficient points to prevent rigid-body motions. This restriction from warping introduces in-plane bending of the flanges called a warping moment or a bimoment. The presence of bimoments induces longitudinal direct stresses (or warping stresses) within the section. This is an important phenomenon and has to be taken into account when analyzing such a class of problem. Ignoring the presence of warping moments can lead to underestimation of stresses and hence, unsafe designs.

A two-member axially discontinuous frame structure is used to demonstrate the ease of analyzing such a problem using a modified one-dimensional element. The frame structure, made up of a typical thin-walled open section, is discretized with three modified beam elements and a joint element. The modified beam element was formulated to model the coupled effect of the warping torsion and the St. Venant torsion whereas the joint element was formulated to model the torsion transfer mechanisms across the angled joint. These formulations were implemented into a standard package called CALFEM and then the structure was analyzed for bending and twisting moments at appropriate sections.

Stresses obtained from the internal forces were used to highlight the importance of incorporating warping effects in the design of thin-walled frame structures, and to illustrate the design procedure which checks the stress levels developed in the structural members. Finally, it is shown that such a simple model formed using beam/joint elements is capable of representing completely the behaviour of thin-walled frame structures with significantly fewer degrees of freedom than that modeled using shell/plate elements.

KEYWORDS. Thin-walled, St. Venant, warping, bimoment, axially-discontinuous, torsion.

INTRODUCTION

Hanizah *et al.* (1996, 1998) presented a simplified analytical procedure that enables the values of the moment and torsion transmitted across the joint of an axially-discontinuous thin-walled frame structure of Figure 1 to be assessed. In order to realize the benefits of the analysis, the results must be used to check the stress levels developed in the members. This paper illustrates the use of the previously obtained analytical results in a design procedure. The design is carried out using the general procedure outlined by Walker (1975) and in accordance with the steel design code BS 5950 Part 5. This states that for members subjected to combined bending and torsion, the maximum stress should be determined on the basis of the full unreduced cross-section and unfactored loads, and should not exceed the design stress p_y.

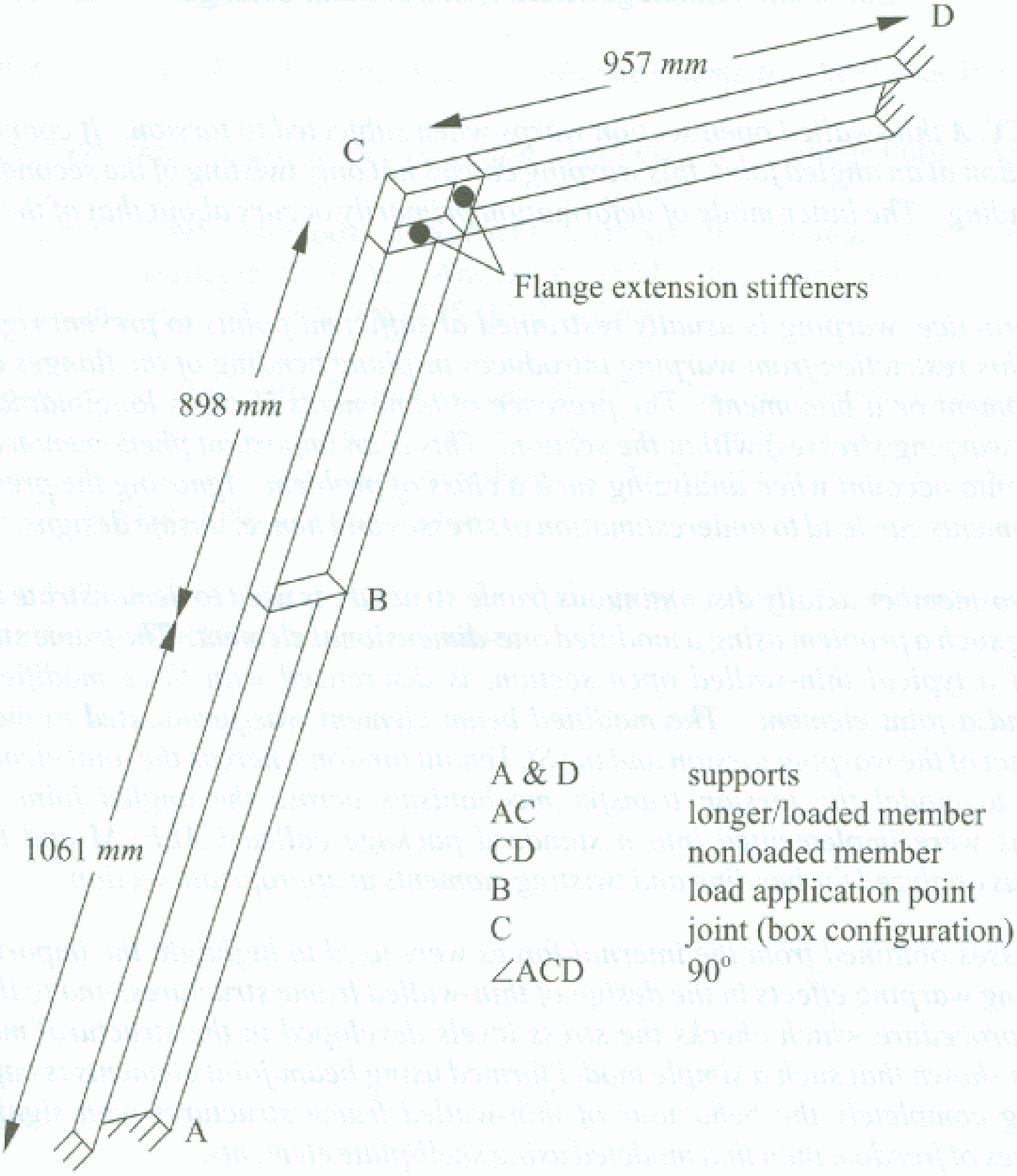


Figure 1. An axially-discontinuous thin-walled frame structure

DESIGN OF MEMBER

The approximate analysis developed in Hanizah *et al*. (1996, 1998) give the values of torsion and moment at the cross-section where the joint is connected to the nonloaded member. This therefore produces the structure shown in Figure 2, which is effectively a single span beam having degrees of fixity at each end, and with point torque and moment applied at the end having the weaker support.

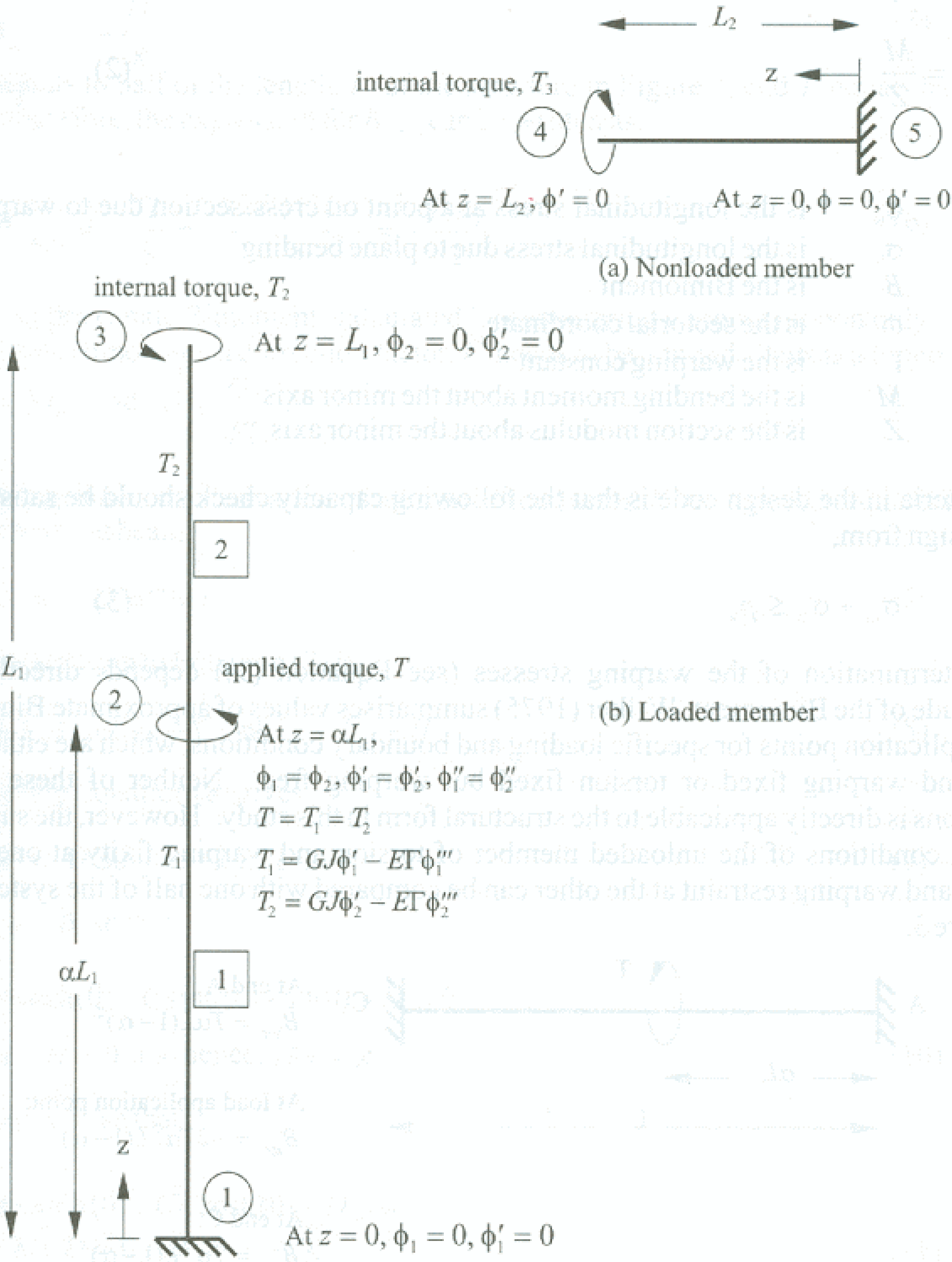


Figure 2. Free-body diagrams

As described in Hanizah *et al.* (1996, 1998), the beam of Figure 2 can neither twist nor warp at the support and the beam is free to twist but not permitted to warp at the other end. Hence, procedures for checking the adequacy of the steel member are governed by the effects of these loading and end conditions which give rise to longitudinal stresses caused by bending moment and warping moment or Bimoment. These stresses can be determined from:

$$\sigma_{\omega} = \frac{B\omega}{\Gamma} \tag{1}$$

$$\sigma_{b} = \frac{M}{Z} \tag{2}$$

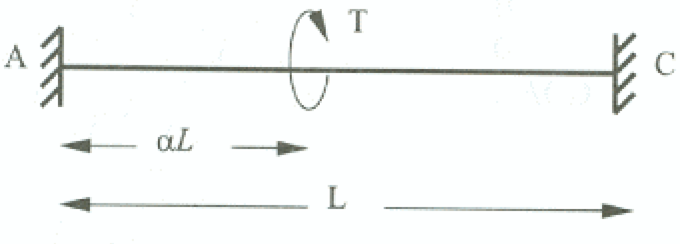
where,

σ_{ω}	is the longitudinal stress at a point on cross-section due to warping
σ_{b}	is the longitudinal stress due to plane bending
B	is the Bimoment
ω	is the sectorial coordinate
Γ	is the warping constant
M	is the bending moment about the minor axis
Z	is the section modulus about the minor axis

The criteria in the design code is that the following capacity check should be satisfied for a safe design from,

$$\sigma_{\omega} + \sigma_{b} \leq p_{y} \tag{3}$$

The determination of the warping stresses (see Equation (1)) depends directly on the magnitude of the Bimoment. Walker (1975) summarises values of approximate Bimoment at load application points for specific loading and boundary conditions, which are either torsion fixed and warping fixed or torsion fixed but warping free. Neither of these idealised conditions is directly applicable to the structural form in this study. However, the support and loading conditions of the unloaded member of torsion and warping fixity at one end and torsion and warping restraint at the other can be compared with one half of the system shown in Figure 3.



At end A:
$B_{app} = T\alpha L(1-\alpha)^2$

At load application point:
$B_{app} = -2T\alpha^2 L(1-\alpha)^2$

At end C:
$B_{app} = T\alpha^2 L(1-\alpha)$

Figure 3. Approximate bimoment, B_{app}, for a fixed-end torque loaded member

Using this figure therefore provides the following formula for bimoment:

At the load application point, the approximate Bimoment, B, is given by,

$$B_{\text{approx}} = -2T\alpha^2 L(1-\alpha)^2 \tag{4}$$

In order to obtain the maximum value, let $\alpha = 0.5$, and B_{approx} reduces to,

$$B_{\text{approx}} = -\frac{TL}{8} \tag{5}$$

Since L_2 equals to half of the length, L, of the structure in Figure 3, and T_3 equals half of the torque, T, therefore, the expression for B_{approx} can be written as,

$$B_{\text{approx}} = -\frac{TL}{8} = -\frac{(2T_3)(2L_2)}{8} = -\frac{T_3 L_2}{2} \tag{6}$$

This is the approximate Bimoment, calculated by considering warping torsion only. In order to find the true Bimoment, a correction factor, F, needs to be applied. This is a function of λL where $L = 2L_2$ and $\lambda^2 = \frac{GJ}{E\Gamma}$.

The following shows the derivation of the correction factor, F, for a point torque at the mid-span of a fixed end beam.

$$GJ\phi' - E\Gamma\phi''' = 0 \tag{7}$$

The general solution to Equation (7) is,

$$\phi = A\cosh\lambda z + C\sinh\lambda z + Dz + H \tag{8}$$

Differentiating Equation (8) gives,

$$\phi' = A\lambda\sinh\lambda z + C\lambda\cosh\lambda z + D \tag{9}$$

At $z = 0, \phi = 0$ so that,

$$A\cosh(0) + C\sinh(0) + D(0) + H = 0$$

$$A + H = 0 \text{ and hence, } H = -A \tag{10}$$

At $z = 0, \phi' = 0$ so that,

$$A\lambda\sinh(0) + C\lambda\cosh(0) + D = 0$$

$$C\lambda + D = 0 \text{ , or } D = -C\lambda \tag{11}$$

Therefore, from Equation (8),

$$\phi = A\cosh\lambda z + C\sinh\lambda z - C\lambda z - A \tag{12a}$$

$$\phi' = A\lambda\sinh\lambda z + C\lambda\cosh\lambda z - C\lambda \tag{12b}$$

$$\phi'' = A\lambda^2\cosh\lambda z + C\lambda^2\sinh\lambda z \tag{12c}$$

$$\phi''' = A\lambda^3\sinh\lambda z + C\lambda^3\cosh\lambda z \tag{12d}$$

At $z = L_2$, $\phi' = 0$, and from Equation (12b),

$$A\lambda\sinh\lambda L_2 + C\lambda\cosh\lambda L_2 - C\lambda = 0$$

or,

$$A = -\frac{C(\cosh\lambda L_2 - 1)}{\sinh\lambda L_2} \tag{13}$$

At $z = L_2$, the total torque to the right of the load is T_3, and since $\phi' = 0$, Equation (7) reduces to,

$$T_3 = -E\Gamma\phi''' \tag{14}$$

Substituting Equation (12d) into (14), and making use of Equation (13) gives the expression for C as,

$$\begin{aligned} T_3 &= -E\Gamma\left(A\lambda^3\sinh\lambda L_2 + C\lambda^3\cosh\lambda L_2\right) \\ &= -E\Gamma\lambda^3\left[-\frac{C(\cosh\lambda L_2 - 1)}{\sinh\lambda L_2}\sinh\lambda L_2 + C\cosh\lambda L_2\right] \\ &= -E\Gamma\lambda^3 C \end{aligned}$$

or,

$$C = -\frac{T_3}{E\Gamma\lambda^3} \tag{15}$$

The true bimoment B_{true} at $z = L_2$ is therefore obtained by making use of Equations (12c), (13), and (15) as,

$$\begin{aligned} B_{true} &= -E\Gamma\phi'' \\ &= -E\Gamma\left(A\lambda^2\cosh\lambda L_2 + C\lambda^2\sinh\lambda L_2\right) \\ &= -E\Gamma C\lambda^2\left[-\frac{(\cosh\lambda L_2 - 1)}{\sinh\lambda L_2}\cosh\lambda L_2 + \sinh\lambda L_2\right] \\ &= \frac{T_3}{\lambda}\frac{(\cosh\lambda L_2 - 1)}{\sinh\lambda L_2} \end{aligned} \tag{16}$$

Equation (16) can also be rewritten in the form of,

$$B_{\text{true}} = B_{\text{approx}}(F) \tag{17}$$

Comparing Equations (6) and (16) allows the correction factor F to be expressed as,

$$F = \frac{B_{\text{true}}}{B_{\text{approx}}}$$

$$= \frac{\frac{T_3}{\lambda}\left(\frac{\cosh \lambda L_2 - 1}{\sinh \lambda L_2}\right)}{\frac{T_3 L_2}{2}}$$

or,

$$F = \frac{2}{\lambda L_2}\left(\frac{\cosh \lambda L_2 - 1}{\sinh \lambda L_2}\right) \tag{18}$$

The graph of the correction factor, F, against beam parameter L_2 is shown in Figure 4.

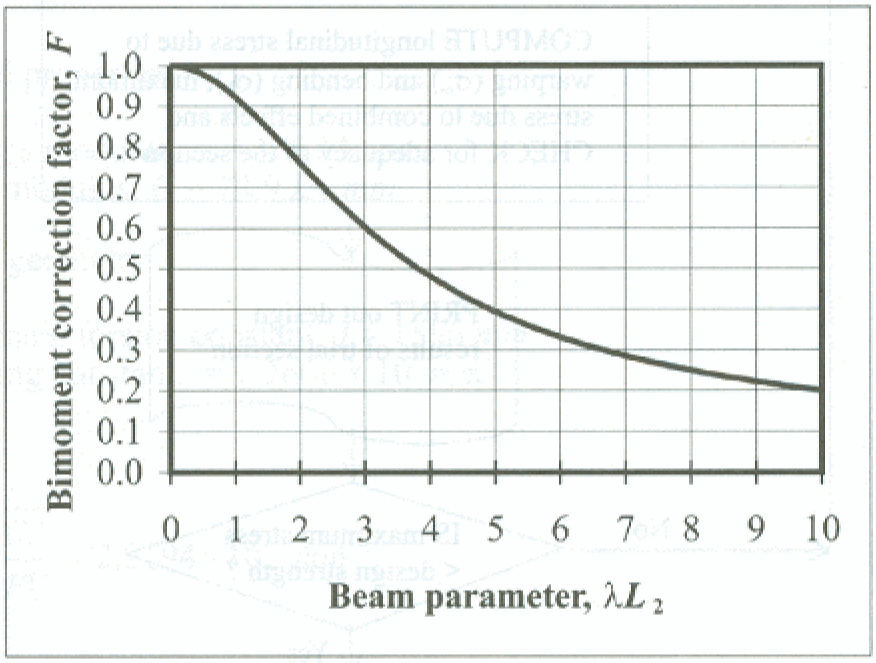


Figure 4. Graph of Bimoment correction factor, F, against Beam parameter, λL_2

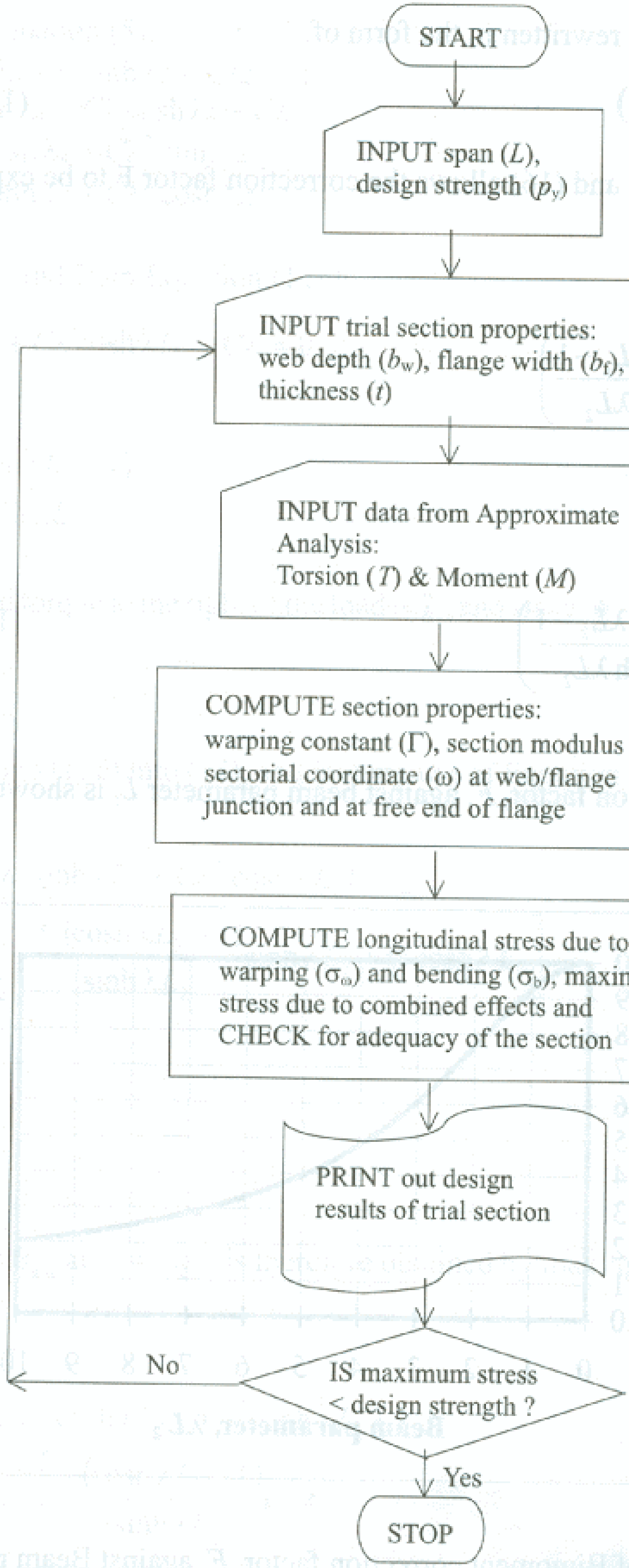


Figure 5. Block flowchart of the design procedure

DESIGN PROCEDURE AND NUMERICAL EXAMPLE

Detailed design consists of selecting suitable member sizes. The strength must be checked to ensure that the cross-section meets all the requirements stated in the appropriate code. The flowchart of the design procedure program is shown in Figure 5.

An example frame is chosen and presented to demonstrate the application of the approximate method. From BS 5950: Part 5, Table 4, the design strength p_y of the material is taken to be 275 N/mm^2. In this case the type of steel is assumed to be equivalent to Steel Grade 43. From Hanizah *et al.*, (1996, 1998), the maximum values of Moment and Torsion of the beam are as follows:

Moment about minor axis, $M = 503.5\ Nm$
Torsion, $T = 161.3\ Nm$
Length, $L = 957\ mm$
Trial plain channel section: 100 x 40 x 3 mm
($b_w = 97 mm$; $b_f = 38.5\ mm$; $t = 3\ mm$)

$$\text{Shear center, } e = \frac{3b_f^2}{6b_f + b_w} = 13.56\ mm$$

The approximate Bimoment (see Equation (6)) for this loading and support conditions is,

$$B_{\text{approx}} = -\frac{T_3 L_2}{2} = -\frac{161.3 \times 10^3 \times 957}{2} = -7.7182 \times 10^7\ Nmm^2$$

From the material properties,

Young's modulus, $E = 188\ kN/mm^2$
shear modulus, $G = 74.9\ kN/mm^2$

From the section geometry,

St. Venant torsion constant, $J = 1566\ mm^4$
Warping constant, $= 1.2666 \times 10^8\ mm^6$

Hence,

$$\lambda = \sqrt{\frac{GJ}{E\Gamma}} = 2.2194 \times 10^{-3}\ mm^{-1}$$

and,

$$\lambda L = 2.2194 \times 10^{-3} \times 957 = 2.124$$

From Equation (18) or the graph in Figure 4, F is obtained as 0.74 so that the true Bimoment B_{true},

$$B_{true} = B_{approx}(F) = -5.7115 \times 10^7 \; Nmm^2$$

The longitudinal stress due to warping , is obtained from Equation (1) as,

$$\sigma_\omega = \frac{B_{true}\omega}{\Gamma}$$

The maximum longitudinal stresses due to warping for a plain channel occurs either at the free edge of the flanges or at the web/flange junctions:

- At the web/flange junctions, $\omega = \dfrac{eb_w}{2} = 657.5 \; mm^2$
- At the free edge of the flanges, $\omega = \dfrac{(b_f - e)b_w}{2} = 1209.7 \; mm^2$

Therefore, the longitudinal stress at the web/flange junctions is,

$$\sigma_\omega = \frac{5.7115 \times 10^7 \times 657.5}{1.2666 \times 10^8} = 296.5 \quad N/mm^2$$

and, the longitudinal stress at the free edge of the flanges is,

$$\sigma_\omega = \frac{5.7115 \times 10^7 \times 1209.7}{1.2666 \times 10^8} = 545.5 \quad N/mm^2$$

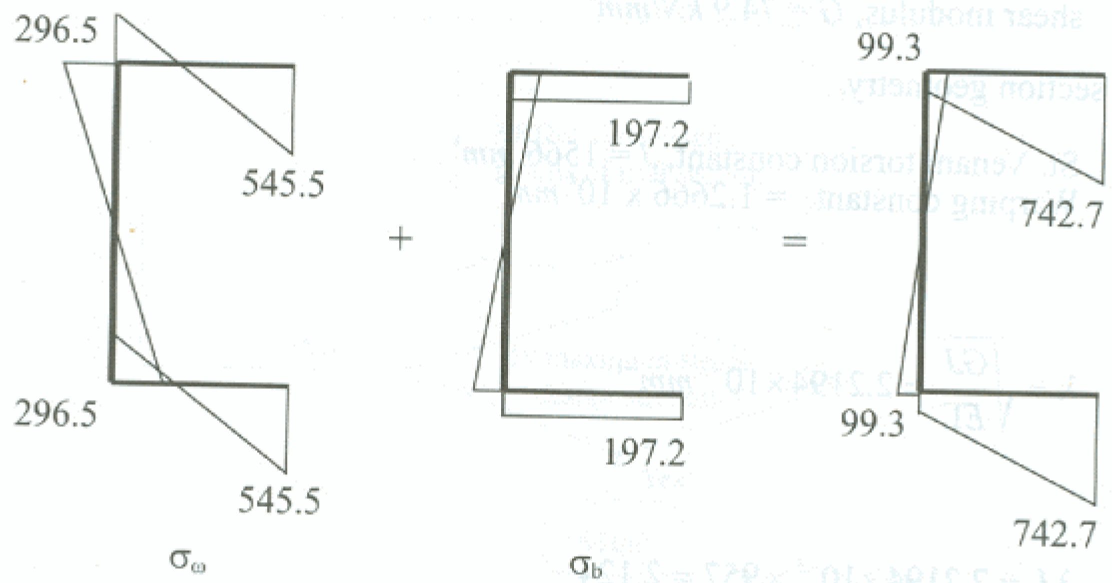


Figure 6. Stress distribution in N/mm^2

The section modulus about the minor axis, $Z = 2553.4\ mm^3$

The maximum longitudinal stress due to bending σ_b is,

$$\sigma_b = \frac{M}{Z} = \frac{503.5 \times 10^3}{2553.4} = 197.2\ N/mm^2$$

The distributions of these stress components around the cross-section are shown in Figure 6. The maximum stress intensity of 742.7 N/mm^2, which occurs at the free edge of the flanges is greater than the permissible design strength, 275 N/mm^2. This implies that the size of the trial cross-section is unsuitable. Hence, a bigger cross-section needs to be selected for the above specified problem. The result from the above example shows that including the warping effects is a necessity. If the effects were not considered, the trial cross-section would have been concluded as safe for its intended use.

CONCLUSION

A design procedure for checking the stress levels developed in the structural member is presented. Hanizah (1994) reported that samples of section sizes, which are not included in this paper, were examined and found to be unacceptable since an allowance is made for torsion and for bending. However, to make a proper comparison, the flexural stresses should be calculated from first principles, i.e. ignoring all torsional effects, calculating the moment in the unloaded member from the end conditions of the loaded member, and then treating the unloaded member as a cantilever section with an end moment.

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